On the Forwarding Area of Contention-Based Geographic Forwarding for Ad Hoc and Sensor Networks

Dazhi Chen
Department of EECS
Syracuse University
Syracuse, NY 13244
dchen02@syr.edu

Jing Deng
Department of CS
University of New Orleans
New Orleans, LA 70148
jing@cs.uno.edu

Pramod K. Varshney
Department of EECS
Syracuse University
Syracuse, NY 13244
varshney@ecs.syr.edu

***Abstract*— Contention-based Geographic Forwarding (CGF) is a state-free communication paradigm for information delivery in multihop ad hoc and sensor networks. *A priori* selection of the forwarding area impacts its overall network performance and the design of the CGF protocol as well. In this work, we study the fundamental problem of defining the forwarding area *a priori* for CGF and determine its impact on the performance. We model CGF without void (i.e., absence of a next-hop node in the forwarding area) handling as a 3-step forwarding strategy. Based on this model and given a random distribution of network nodes, we develop a general mathematical analysis technique to evaluate the performance of CGF with different forwarding areas, in terms of the performance metric *average single-hop packet progress*. Further, we introduce two state-free void handling schemes, i.e., *active exploration* and *passive participation*, for CGF and study their performance in depth. Our theoretical analysis and numerically evaluated results, validated by extensive simulations, provide a guideline regarding the selection of specific forwarding areas for the design of a practical CGF protocol. It also serves as a general performance evaluation framework for the existing CGF protocols.**

I. Introduction

In recent years, a communication paradigm, based on the distributed selection of a next-hop node via contention, has been proposed for information delivery in multi-hop wireless ad hoc and sensor networks [1]–[5]. In this paradigm, the geographic location information of the nodes is assumed available, either by *a priori* configuration or by the use of the Global Positioning System (GPS) receiver or through self-configuring localization mechanisms such as [6], [7]. We call this paradigm *Contention-based Geographic Forwarding (CGF)*.

Different from the topology-based communication paradigm, which requires a routing protocol such as Dynamic Source Routing (DSR) [8] to collect the topology information, CGF takes advantage of the geographic information to allow the data packets to gradually approach and eventually arrive at the intended destination. Compared with the traditional geographic communication paradigm, CGF allows next-hop candidate nodes to contend for the data forwarding task and a specific next-hop node is decided at the transmission time. The contention mechanism eliminates the need to periodically transmit beacons, which are used to update the location information of neighbors, and the need to select a next-hop node in advance at the sender. Thus, nodes employing CGF only need to know their own locations and that of the packet destination[1] in order to communicate with each other. Furthermore, the centralized selection of a next-hop node at the sender is replaced by the distributed selection of a next-hop node among the potential receivers.

Since CGF requires neither the topology information nor the neighbor location information, it is regarded as a *state-free* communication paradigm. Its state-free nature makes CGF robust to high network dynamics and frequent topology changes, scalable to large-scale node deployment, adaptable to dense node densities, and applicable to data-centric applications and resource-constrained networks. Furthermore, contention among the potential receivers does not require significantly more

[1]A source node can obtain the location information of a data packet destination either from a location service such as geographical location service (GLS) [9] or via *a priori* configuration. This information is encoded into the data packet header. Any intermediate nodes can directly access it later.

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overhead than contention resolution at the MAC layer. Due to these benefits, CGF has recently attracted a significant interest from the research community as a favored candidate for information delivery in Vehicular Ad Hoc Networks (VANETs) [4], Mobile Ad Hoc Networks (MANETs) [1], Wireless Sensor Networks (WSNs) [2], [3], [5], and Wireless Sensor and Actuator Networks (WSANs).

While different CGF protocols adopt different approaches [1]–[5], the basic idea remains the same given as follows: i) a predefined forwarding area and nodes that reside in the area become next-hop candidate nodes and contend with one another for the data forwarding task; ii) a distributed contention arbitration and resolution scheme in order to effectively establish a single next-hop node in the forwarding area; iii) the establishment of next-hop node selection criteria in order to attain the desired network performance in an efficient way; iv) an effective mechanism to handle a void.[2]

Except the GeRaF protocol that uses an area-based and sender-assisted contention arbitration and resolution mechanism [2], all other CGF protocols employ a timer-based and receiver-overheard contention arbitration and resolution mechanism [1], [3]–[5]. One of the basic processes in the schemes proposed in [1], [3]–[5] is the following: first, the sender broadcasts an RTS packet to all its neighbors. Second, the neighbors residing in the forwarding area independently set up timers in order to contend for the data forwarding task. A candidate node that has the earliest timeout is arbitrated to win the contention. Third, the winning node returns a CTS packet to establish itself as the only next-hop node, other nodes in the forwarding area that overhear the CTS packet drop out of the contention. Finally, a normal DATA and ACK exchange between two specific nodes follows. The above CGF process can be easily integrated into a contention-based MAC protocol such as the IEEE 802.11 Distributed Coordinator Function (DCF) [11]. Such cross-layer design reduces the protocol overhead and the data forwarding delay [3], [5].

A fundamental problem in CGF is the definition of the forwarding area, since it influences how much progress a packet makes towards a destination on a single hop and thereby has a significant impact on the overall network performance of CGF. The forwarding area also affects the design of the contention resolution mechanism and the possibility of having a void, the event that no next-hop candidate nodes are present in the forwarding area.

In this work, we approach the problem of defining the forwarding area for CGF analytically and through extensive simulations. We concentrate on CGF in wireless ad hoc networks with randomly distributed nodes, where any pair of nodes may communicate with each other, either directly or through multiple hops.[3] To the best of our knowledge, our work is the first study of CGF from this perspective. The main contributions of this work include:

- A high-level model of CGF without void handling is established as a 3-step forwarding strategy. This model can be used to analyze different geography-based protocols from a number of perspectives such as evaluation of multi-hop performance and energy efficiency;
- A general mathematical analysis framework, in terms of *average single-hop packet progress*, is developed for the performance evaluation of CGF with most regular forwarding areas of interest;
- The conclusion that, maximum forwarding area, at the cost of more complex protocol design and lower spatial reuse, always performs the best. Also, a 60-degree radian area begins to achieve the same performance as maximum forwarding area with an average number of neighbors equal to 40 while maximum communication area requires much more than 100 neighbors on an average to do so.[4] This result is validated by extensive simulations and is useful for the design of a practical CGF protocol;
- The introduction of two state-free void handling schemes for CGF and their performance evaluation via extensive simulations.

The rest of the paper is organized as follows. In Section II, we define and discuss three typical forwarding areas for CGF that have been adopted in the literature [1]–[5]. A high-level abstraction of CGF as a forwarding strategy and a theoretical analysis for the performance of CGF are presented in Section III. Section IV provides numerical and simulation results along with some discussion. In Section V, we introduce two state-free void handling schemes and investigate their performance. We conclude our work in Section VI.

[2]The absence of a next-hop node in the forwarding area. This problem is also termed as *holes* or *local minimum phenomenon* in the literature [10].

[3]While, in the rest of this paper, we focus our attention on wireless ad hoc networks, the results obtained are also applicable to wireless sensor networks, where the information sink may be at any location in the region of interest.

[4]Definitions for these forwarding areas will be given in Section II.

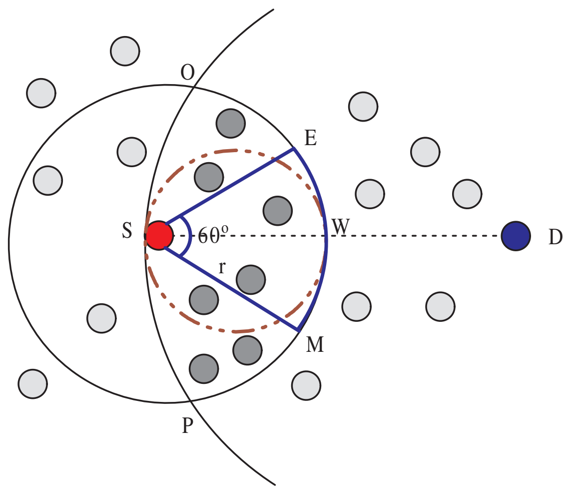


Fig. 1. Three Typical Forwarding Areas for CGF

II. Forwarding Areas in CGF

A. Forwarding Areas

With a common requirement among all forwarding areas that a packet should make a positive progress toward the destination at every hop and an assumption that each node has a transmission range in the form of a circle, we define three typical forwarding areas, as shown in Fig. 1, for CGF as follows:

- **Maximum Forwarding Area (MFA)**: MFA is the overlap region of two circular areas: the transmission circle of the sender and the circle that is centered at the destination with a radius equal to the distance between the sender and the destination. It is the region OSPW shown in Fig. 1. The size of this area is variable and dependent on the distance between the sender and the destination. Note that this forwarding area has been adopted in the GeRaF [2] and SIF [5] protocols.
- **Maximum Communication Area (MCA)**: MCA is defined as the largest region within which any pair of nodes can hear each other [1]. Thus, it is a circle with a diameter equal to the transmission range of a node. It is the circle with SW as its diameter shown in Fig. 1. The beauty of MCA is that, as suggested by its name, this is the maximum area within which every pair of nodes can hear each other, eliminating potential CTS response collisions unless the responses are sent at the same time. Note that SW should be co-linear with SD in order to maximize the possible area within the circle. The size of MCA is a constant no matter where the destination is located.
- **60-Degree Radian Area (DRA)**: DRA is a radial region that includes a 30-degree radian area around the line connecting the sender and the destination on both sides. It is the region SEWM shown in Fig. 1. Note that SD is the angle bisector for this region. The size of DRA is also a constant. DRA was used in the IGF protocol [3].

As we mentioned earlier, a timer-based and receiver-overheard CGF protocol requires that any pair of nodes in a forwarding area should be able to hear each other in order to avoid multiple responses. The MCA and DRA areas satisfy such a requirement as all points in these regions are within the communication range of each other. The MFA area, however, may have two nodes that are out of their communication ranges. Thus, it is possible for two or more next-hop candidate nodes to collide during the contention process. This issue was addressed by a technique that exploits the difference between carrier sensing range and transmission range in [5].[5] We do not consider the collisions from the next-hop candidate nodes and the associated additional cost for MFA in this work.

Since the size of MFA is larger than those of MCA and DRA, the probability of void in MFA is smallest among the three areas, based on the assumption of random deployment of network nodes. These probabilities are calculated in the following subsection.

B. Probability of void

We model the node distribution in the region of interest as a two-dimensional Poisson point process. Thus, the probability that k nodes are located within an area of size A is given by [12]:

$$\Pr\{k\} = \frac{(\lambda A)^k \cdot e^{-\lambda A}}{k!} , \quad (1)$$

where λ is the expected number of nodes within a unit area.

Since the forwarding areas of MCA and DRA are, $\pi r^2/4$ and $\pi r^2/6$, respectively, the probabilities of void ($\Pr\{k=0\}$) for MCA and DRA are

$$\Pr\{\text{void in MCA}\} = e^{-\frac{\rho}{4}} , \quad (2)$$
$$\Pr\{\text{void in DRA}\} = e^{-\frac{\rho}{6}} , \quad (3)$$

where ρ is the average number of neighbors within the transmission range r of the sender and is given by

$$\rho = \pi r^2 \lambda . \quad (4)$$

[5]Note that, such a technique requires a more complex protocol design and reduces the spatial reuse at the MAC layer as well.

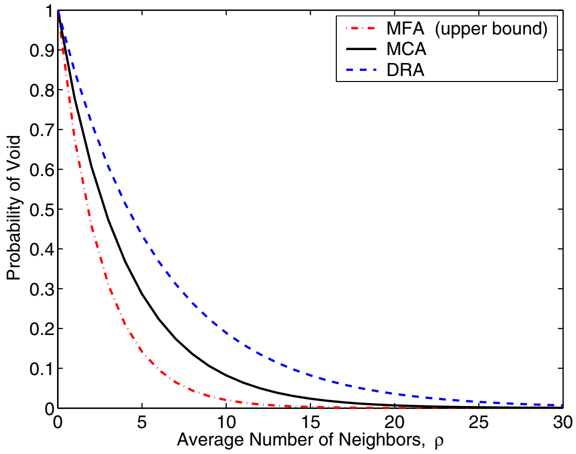


Fig. 2. Probability of Void for Forwarding Areas

The calculation of the probability of void for the MFA area is more complicated. However, an upper bound has been derived in [5] as

$$\Pr\{\text{void in MFA}\} \quad \leq \quad e^{-(\frac{2}{3}-\frac{\sqrt{3}}{2\pi})\rho} \approx e^{-\frac{\rho}{2.5}} \ . \qquad (5)$$

The results from (2), (3), and (5) are plotted in Fig. 2. From the figure, it is observed that the upper bound on the probability of void of MFA is lower than the probability of void of MCA and DRA. Note that, when ρ is higher than 30, probabilities of void for all three areas are close to zero. This suggests that at least one next-hop node will be found in all three forwarding areas.

Although MFA has a lower probability of void than MCA and DRA, a more interesting and challenging issue is: does CGF that employs MFA perform *better* than CGF that uses MCA or DRA, in a randomly deployed network? What metrics can be used to evaluate them? In the next section, we will propose a performance metric and present a theoretical analysis.

III. Performance Analysis

A. Performance Metric

In order to evaluate different CGF protocols, we propose to use the metric *average single-hop packet progress*, which is defined as the expected value of the difference between the before-hop distance (between the sender and the destination) and the after-hop distance (between the next-hop node and the destination).[6] Such a metric reflects the progress toward the intended destination at each hop and has a direct connection with the end-to-end packet delay performance of CGF in the long term. In the following subsections, we will derive average single-hop packet progress for randomly deployed networks that employ CGF with the MFA, MCA, and DRA forwarding areas, respectively.

B. Network Assumptions and Abstract CGF Strategy

We assume that a homogeneous wireless ad hoc network is deployed with randomly distributed nodes, where all nodes have the common transmission range in the form of a circle of radius r. Nodes adopt the CGF protocol to communicate with each other and the forwarding area in CGF is specified *a priori*. The node distribution can be modeled as in Equation (1) and the appearance of nodes in any two non-overlapping areas is independent.

Due to the limited radio range, a data packet has to be forwarded by a certain number of hops via intermediate nodes before reaching its destination. For any single hop, a node within the forwarding area of a sender is called *a next-hop candidate node* for the sender. We assume a perfect contention arbitration and resolution scheme for CGF, which makes a candidate node closest to the destination in the forwarding area always win the contention[7] and become the only next-hop node for the sender. We also assume the availability of a collision-free media access scheme and that each node can determine its own location and the location of packet destination.

Further, we consider the snapshot at the time of the *first-hop* packet forwarding, even if a multihop forwarding is required for most data packets to arrive at their destinations. Given a random node deployment, any intermediate-hop forwarding can be viewed as a new *first-hop* packet forwarding. Thus, the first-hop average packet progress is close to the overall average single-hop packet progress in a homogeneous environment. We assume that a source node, S, is located at the center of a circle of radius x, where x is termed the *network range* and is the largest possible distance between S and any destination. In other words, the source node will not send any packet to nodes outside the circle. The destination node D, to which S intends to deliver a data packet, is assumed to be randomly distributed over the entire circle.

[6]The definition of progress in this paper is the same as that of advance in [13]. However, note that progress is originally defined, in the literature [13], as the orthogonal projection of the line connecting the sender and the next-hop node onto the line connecting the sender and the destination. It differs from our definition.

[7]Note that other criteria for a distributed selection of a next-hop node may be used, such as energy [3], [5] and link reliability. In this work, we focus on a distance-only criterion.

Based on the above assumptions, CGF can be modeled as a 3-step forwarding strategy given below:

(i) If the destination node D is located within distance r from the source node S, D always wins the contention and the source node S delivers a data packet to D directly.
(ii) If the destination node D is outside the transmission range of the source node S, the data packet is always forwarded by a next-hop node that is closest to the destination D in the forwarding area.
(iii) If there are no next-hop candidate nodes in the forwarding area, the source node S does not forward the data packet and discards it after a pre-determined period of time during which no candidate nodes appear in the forwarding area.[8]

C. Average Single-hop Packet Progress

First of all, we derive a general expression for average first-hop packet progress, which has been assumed to be approximately equal to the average single-hop packet progress under our assumptions, for CGF. Let v denote the distance between the source node S and the destination node D. When $v \leq r$, direct data delivery to the destination node D can be completed, according to step (i) in the forwarding strategy; hence, the data packet progress toward the destination node D is v. If $v > r$, an appropriate (i.e., nearest to the destination in this context, according to step (ii) in the forwarding strategy) next-hop node in its forwarding area is selected for further packet forwarding. The data packet progress toward the destination node D is thereby equal to $(v - n)$, where n is the distance between the next-hop node F and the destination node D corresponding to the first hop.

Denote by $\boldsymbol{P}$ the random variable corresponding to the data packet progress in a single hop. Let $\boldsymbol{V}$ and $\boldsymbol{N}$, respectively, be the random variables corresponding to v and n discussed above. $\boldsymbol{I}$ is an index random variable defined as below:

$$\boldsymbol{I} = \begin{cases} 1, & \text{at least one node in the forwarding area;} \\ 0, & \text{otherwise.} \end{cases}$$

With the above notations, the problem of finding the average single-hop packet progress is equivalent to the

[8] In this work, we first consider CGF without void handling since void handling complicates and hides the performance of a CGF protocol. We will investigate void handling schemes in Section V.

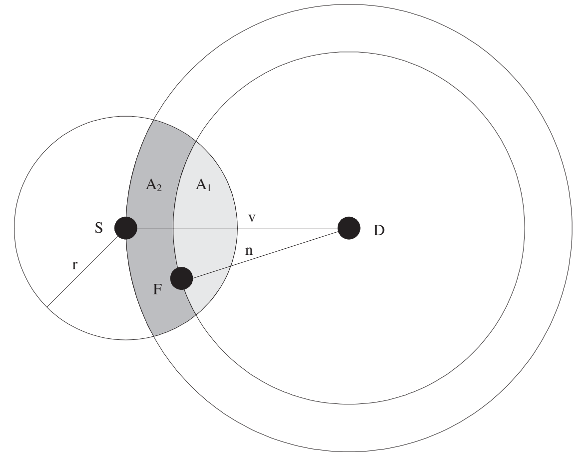


Fig. 3. Illustration of Maximum Forwarding Area (MFA)

derivation of the expected value of $\boldsymbol{P}$, where

$$\boldsymbol{P} = \begin{cases} \boldsymbol{V}, & \text{if } \boldsymbol{V} \leq r; \\ \boldsymbol{V} - \boldsymbol{N}, & \text{if } \boldsymbol{V} > r \cap \boldsymbol{I} = 1; \\ 0, & \text{if } \boldsymbol{V} > r \cap \boldsymbol{I} = 0. \end{cases} \quad (6)$$

Note that, if $(\boldsymbol{V} > r \cap \boldsymbol{I} = 0)$ is true, no packet forwarding will occur. In this case, the actual packet progress is zero. It can, therefore, be verified that

$$E[\boldsymbol{P}] = E[\boldsymbol{P} | (\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)] \Pr\{(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)\}. \quad (7)$$

We now proceed to derive the expectation of $\boldsymbol{P}$ given that $(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)$ is true.[9] From [14], we have:

$$\Pr\{\boldsymbol{P} > p | (\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)\} = \frac{\Pr\{p < \boldsymbol{V} \leq r\} + \Pr\{\boldsymbol{V} - \boldsymbol{N} > p \cap \boldsymbol{V} > r \cap \boldsymbol{I} = 1\}}{\Pr\{\boldsymbol{V} \leq r\} + \Pr\{\boldsymbol{V} > r \cap \boldsymbol{I} = 1\}}. \quad (8)$$

We therefore need to determine four probability expressions in (8). Among them, $\Pr\{p < \boldsymbol{V} \leq r\}$ and $\Pr\{\boldsymbol{V} \leq r\}$ can be easily determined below:

$$\Pr\{p < \boldsymbol{V} \leq r\} = \frac{(r^2 - p^2)}{x^2} \boldsymbol{U}(r - p), \quad (9)$$

$$\Pr\{\boldsymbol{V} \leq r\} = \frac{r^2}{x^2}, \quad (10)$$

[9] The derivation of this conditional expectation from (8) to (15) is similar to that given in [14], where the authors used such a progress to derive an optimum node transmission range for ad hoc networks. In this work, we further develop it as a part of our generalized evaluation framework for CGF, independent of any forwarding areas.

where $\boldsymbol{U}$ function is defined as:

$$\boldsymbol{U}(r-p) = \begin{cases} 1, & \text{if } r \geq p; \\ 0, & \text{if } r < p. \end{cases}$$

It remains to determine $\Pr\{\boldsymbol{V} > r \cap \boldsymbol{I} = 1\}$ and $\Pr\{\boldsymbol{V} - \boldsymbol{N} > p \cap \boldsymbol{V} > r \cap \boldsymbol{I} = 1\}$. Next, we proceed to determine $\Pr\{\boldsymbol{V} > r \cap \boldsymbol{I} = 1\}$.

Let A_{SD} denote a general forwarding area. Further, we can divide A_{SD} into two regions by the circle centered at D with radius n. The areas of these two regions are respectively denoted as A_1 and A_2, as the shaded regions shown in Fig. 3, Fig. 4, and Fig. 5.

$$\begin{aligned}
&\Pr\{\boldsymbol{V} > r \cap \boldsymbol{I} = 1\} \\
&= \int_0^x \Pr\{\boldsymbol{V} > r \cap \boldsymbol{I} = 1 | \boldsymbol{V} = v\} dP_{\boldsymbol{V}}(v) \\
&= \int_r^x \Pr\{\text{at least one node in } A_{SD}\} dP_{\boldsymbol{V}}(v) \quad (11) \\
&= \int_r^x \left(1 - e^{-\lambda A_{SD}}\right) \frac{2v}{x^2} dv \\
&= 1 - \frac{r^2}{x^2} - \frac{2}{x^2} \int_r^x v e^{-\lambda A_{SD}} dv, \quad (12)
\end{aligned}$$

where the lower integration limit is replaced with r in (11) due to $\boldsymbol{V} > r$. So far, we have completed the determination of $\Pr\{\boldsymbol{V} > r \cap \boldsymbol{I} = 1\}$. Further, observe that:

$$\begin{aligned}
&\Pr\{\boldsymbol{N} \geq n \cap \boldsymbol{I} = 1 | \boldsymbol{V} = v\} \\
&= \begin{cases} \Pr\{\boldsymbol{I} = 1 | \boldsymbol{V} = v\}, & \text{if } n \leq v - r; \\ \Pr\{\text{Q}\}, & \text{if } v - r < n < v; \\ 0, & \text{if } n \geq v, \end{cases}
\end{aligned}$$

where Q represents the event in which no neighbors exist in A_1 and at least one exists in A_2. By the assumption of independence of node appearance in non-overlapping regions, the above expression for $v - r < n < v$ can be further derived as:

$$\begin{aligned}
&\Pr\{\text{Q}\} \\
&= \Pr\{\text{no neighbors in } A_1 \text{ and at least one in } A_2\} \\
&= \Pr\{\text{no neighbors in } A_1\} \Pr\{\text{at least one in } A_2\} \\
&= e^{-\lambda A_1}\left(1 - e^{-\rho A_2}\right) \\
&= e^{-\lambda A_1} - e^{-\lambda A_{SD}} \quad .
\end{aligned}$$

As a result,

$$\begin{aligned}
&\Pr\{\boldsymbol{N} < n \cap \boldsymbol{I} = 1 | \boldsymbol{V} = v\} \\
&= \Pr\{\boldsymbol{I} = 1 | \boldsymbol{V} = v\} - \Pr\{N \geq n \cap \boldsymbol{I} = 1 | \boldsymbol{V} = v\} \\
&= \begin{cases} 0, & \text{if } n \leq v - r; \\ 1 - e^{-\lambda A_1}, & \text{if } v - r < n < v; \\ 1 - e^{-\lambda A_{SD}}, & \text{if } n \geq v, \end{cases} \quad (13)
\end{aligned}$$

where $\Pr\{\boldsymbol{I} = 1 | \boldsymbol{V} = v\} = 1 - e^{-\lambda A_{SD}}$ has been used.

Using (13), we obtain:

$$\begin{aligned}
&\Pr\{\boldsymbol{V} - \boldsymbol{N} > p \cap \boldsymbol{V} > r \cap \boldsymbol{I} = 1\} \\
&= \int_r^x \Pr\{\boldsymbol{N} < v - p \cap \boldsymbol{I} = 1 | \boldsymbol{V} = v\} dP_{\boldsymbol{V}}(v) \\
&= \int_r^x \Pr\{\boldsymbol{N} < v - p \cap \boldsymbol{I} = 1 | \boldsymbol{V} = v\} \frac{2v}{x^2} dv \\
&= \left(1 - \frac{r^2}{x^2} - \frac{2}{x^2} \int_r^x v e^{-\lambda A_1} dv\right) \boldsymbol{U}(r-p) \ . \quad (14)
\end{aligned}$$

This determines $\Pr\{\boldsymbol{V} - \boldsymbol{N} > p \cap \boldsymbol{V} > r \cap \boldsymbol{I} = 1\}$. Note that n in (13) has been substituted by $v - p$ during the above derivation. Thus, any n associated with the expression of the A_1 area should be replaced with $v - p$ whenever necessary later.

Finally, substituting (9), (10), (12), and (14) into (8), we have, for $p > 0$,

$$\begin{aligned}
&\Pr\{\boldsymbol{P} > p \,|(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)\} \\
&= \left(\frac{x^2 - p^2 - 2\int_r^x v e^{-\lambda A_1} dv}{x^2 - 2\int_r^x v e^{-\lambda A_{SD}} dv} \right) \boldsymbol{U}(r-p) \ .
\end{aligned}$$

The expected value of $\boldsymbol{P}$ given the validity of $(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)$ is then equal to [15, Eq. (21.9)]:

$$\begin{aligned}
&E\left[\boldsymbol{P} \,|(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)\,\right] \\
&= \int_0^\infty \Pr\{\boldsymbol{P} > p \,|(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)\} dp \\
&= \frac{\int_0^r \left[x^2 - p^2 - 2\int_r^x v e^{-\lambda A_1} dv\right] dp}{x^2 - 2\int_r^x v e^{-\lambda A_{SD}} dv} \\
&= \frac{3x^2 r - r^3 - 6\int_0^r \int_r^x v e^{-\lambda A_1} dv dp}{3\left(x^2 - 2\int_r^x v e^{-\lambda A_{SD}} dv\right)}. \quad (15)
\end{aligned}$$

Substituting (10), (12), and (15) into (7), we attain a general expression for average single-hop data packet progress, independent of any forwarding areas, for networks with CGF below:

$$E[\boldsymbol{P}] = \frac{3x^2 r - r^3 - 6\int_0^r \int_r^x v e^{-\lambda A_1} dv dp}{3x^2}, \quad (16)$$

where we have used

$$\begin{aligned}
&\Pr\{(\boldsymbol{V} \leq r) \cup (\boldsymbol{V} > r \cap \boldsymbol{I} = 1)\} \\
&= \Pr\{\boldsymbol{V} \leq r\} + \Pr\{\boldsymbol{V} > r \cap \boldsymbol{I} = 1\}
\end{aligned}$$

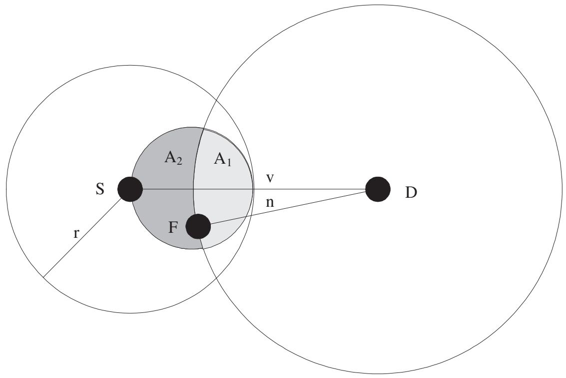


Fig. 4. Illustration of Maximum Communication Area (MCA)

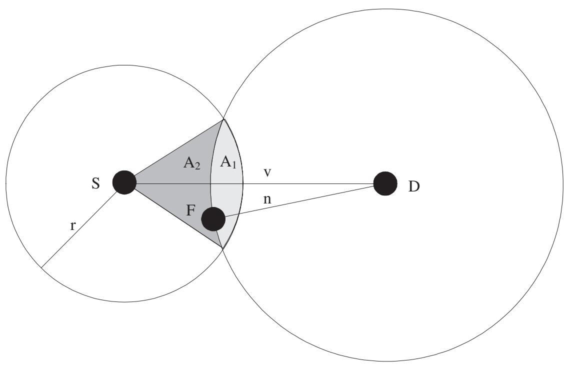


Fig. 5. Illustration of 60-Degree Radian Area (DRA)

since these two events are mutually exclusive.

Now we proceed to derive the average single-hop progress of three different forwarding areas: MFA as shown in Fig. 3, MCA as shown Fig. 4, and DRA as shown in Fig. 5, since A_1 is different for MFA, MCA and DRA.

In the case of MFA, $A_1(MFA)$ is

$$A_1(MFA) = r^2\cos^{-1}\left(\frac{r^2+v^2-n^2}{2rv}\right) + n^2\cos^{-1}\left(\frac{n^2+v^2-r^2}{2vn}\right) - \frac{1}{2}\sqrt{(r+v+n)(v+n-r)(r+v-n)(r+n-v)}\ .$$

Thus, average single-hop packet progress for MFA can be finally determined as:

$$E_{MFA}[\boldsymbol{P}] = \frac{3x^2r - r^3 - 6\int_0^r\int_r^x ve^{-\frac{\rho A_1(MFA)}{\pi r^2}}dvdp}{3x^2}\ . \quad (17)$$

where ρ is given by (4).

In the case of MCA, $A_1(MCA)$ is

$$A_1(MCA) = \begin{cases} A_{11}(MCA) & \text{if } 0 < p < c_1; \\ A_{12}(MCA) & \text{if } c_1 < p < r, \end{cases}$$

where $c_1 = v - \sqrt{(v-\frac{r}{2})^2 + (\frac{r}{2})^2}$ and

$$A_{11}(MCA) = n^2\cos^{-1}\left(\frac{n^2+(v-\frac{r}{2})^2-(\frac{r}{2})^2}{2n(v-\frac{r}{2})}\right) - \frac{1}{2}n^2\sin\left(2\cos^{-1}\left(\frac{n^2+(v-\frac{r}{2})^2-(\frac{r}{2})^2}{2n(v-\frac{r}{2})}\right)\right) + \frac{\pi r^2}{4} - \frac{r^2}{4}\left(\pi - \cos^{-1}\left(\frac{(\frac{r}{2})^2+(v-\frac{r}{2})^2-n^2}{r(v-\frac{r}{2})}\right)\right) + \frac{r^2}{8}\sin\left(2\left(\pi - \cos^{-1}\left(\frac{(\frac{r}{2})^2+(v-\frac{r}{2})^2-n^2}{r(v-\frac{r}{2})}\right)\right)\right)\ ,$$

$$A_{12}(MCA) = n^2\cos^{-1}\left(\frac{n^2+(v-\frac{r}{2})^2-(\frac{r}{2})^2}{2n(v-\frac{r}{2})}\right) - \frac{1}{2}n^2\sin\left(2\cos^{-1}\left(\frac{n^2+(v-\frac{r}{2})^2-(\frac{r}{2})^2}{2n(v-\frac{r}{2})}\right)\right) + \frac{r^2}{4}\cos^{-1}\left(\frac{(\frac{r}{2})^2+(v-\frac{r}{2})^2-n^2}{r(v-\frac{r}{2})}\right) - \frac{r^2}{8}\sin\left(2\cos^{-1}\left(\frac{(\frac{r}{2})^2+(v-\frac{r}{2})^2-n^2}{r(v-\frac{r}{2})}\right)\right)\ .$$

Therefore, average single-hop packet progress for MCA is

$$E_{MCA}[\boldsymbol{P}] = \frac{3x^2r - r^3}{3x^2} - \frac{6\int_r^x vdv\left(\int_0^{c_1} e^{-\frac{\rho A_{11}(MCA)}{\pi r^2}}dp + \int_{c_1}^r e^{-\frac{\rho A_{12}(MCA)}{\pi r^2}}dp\right)}{3x^2}\ . \quad (18)$$

In the case of DRA, $A_1(DRA)$ is

$$A_1(DRA) = \begin{cases} A(DRA) & \text{if } 0 < p < c_2; \\ A_1(MFA) & \text{if } c_2 < p < r, \end{cases}$$

where $c_2 = v - \sqrt{r^2+v^2-\sqrt{3}rv}$ and

$$A(DRA) = n^2\left(\sin^{-1}(\frac{v}{2n} - \frac{\pi}{6})\right) + \frac{\pi r^2}{6} - \sqrt{3}n^2\sin^2\left(\sin^{-1}(\frac{v}{2n} - \frac{\pi}{6})\right) - \frac{1}{2}n^2\sin\left(2\sin^{-1}(\frac{v}{2n} - \frac{\pi}{3})\right)\ .$$

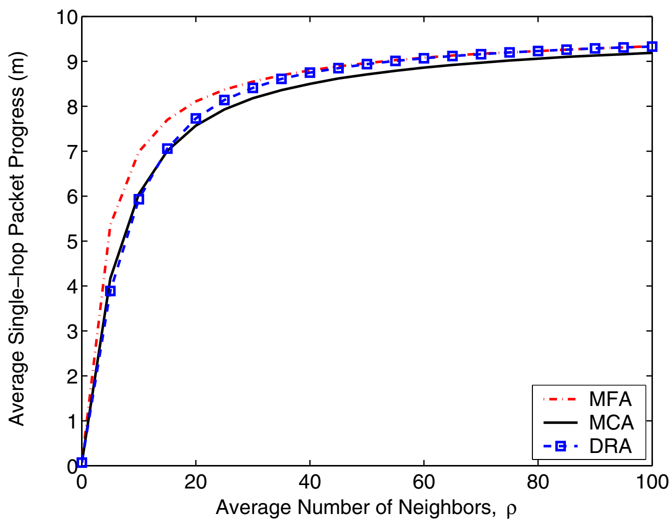


Fig. 6. Numerical results for average single-hop packet progress vs average number of neighbors (r = 10m and x = 100m)

Hence, average single-hop packet progress for DRA can be finally derived as:

$$E_{DRA}[\boldsymbol{P}] = \frac{3x^2 r - r^3}{3x^2} - \frac{6\int_r^x v dv \left(\int_0^{c_2} e^{-\frac{\rho A(DRA)}{\pi r^2}} dp + \int_{c_2}^r e^{-\frac{\rho A_1(MFA)}{\pi r^2}} dp \right)}{3x^2} . \tag{19}$$

IV. Numerical and Simulation Results

A. Numerical Results

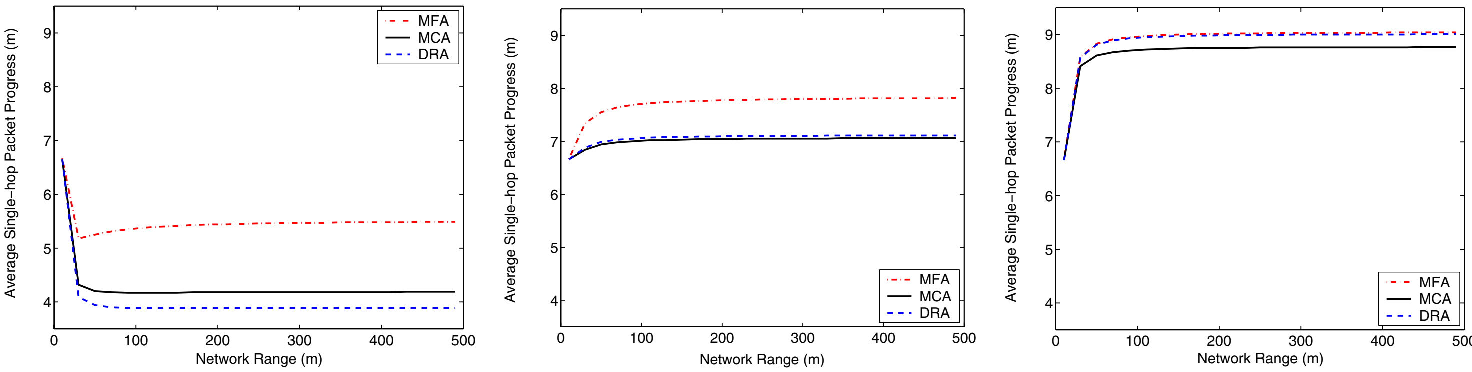
As shown in (17), (18), and (19), the average single-hop packet progress is a function of node transmission range r, network range x, and average number of neighbors ρ (or nodal density λ). We present two sets of numerical results as shown in Figs. 6 and 7. Figure 6 compares the analytically obtained average single-hop packet progress for MFA, MCA, and DRA, with respect to different average number of neighbors. The network range is fixed to be a circle with radius 100 meters (i.e., $x = 100m$) while node transmission range is set at $10m$ (i.e., $r = 10m$). Note that similar results have been observed for other values of r and are not presented here.

It can be observed from Fig 6 that the average single-hop packet progress approaches r with an increase in ρ, for all three techniques. However, it does not become equal to 0 or r. This is expected: when ρ is very small, i.e., close to 0, it is unlikely that the sender will find any next-hop candidate to forward its data packets. However, it is still possible that the sender and the destination are within the communication range of each other. Thus, the average single-hop packet progress is positive. When ρ increases, the probability of void decreases and the sender is able to locate a next-hop node in its forwarding area in most situations. The average single-hop packet progress, therefore, rises. As ρ becomes larger and larger, it is more and more likely for the sender to locate a next-hop node nearest to the destination and farthest away from the sender as well, i.e., the data packet progress during this hop is equal to node transmission range r. As a result, we can expect that, when ρ goes to infinity, average single-hop packet progress will approach r asymptotically. However, it is still possible for a destination node to be within the range of the sender, then the packet progress is less than or equal to r. Therefore, the average single-hop packet progress will not become equal to r.

Another interesting observation from Fig. 6 is that the average single-hop packet progress increases quickly for small ρ and then slowly after ρ exceeds a certain threshold value, i.e., there exists a knee in the curves around $\rho \approx 10-20$. The explanation of this phenomenon lies in the probability of void as shown in Fig. 2. From Fig. 2, we can see that the probability of void decreases quickly as ρ increases from 0 to the above threshold value of ρ. This observation suggests that the probability of void dominates the average single-hop packet progress, especially when ρ is small and the network is sparse. As a result, the trend of the curves in Fig. 6 is inverse of that of the curves in Fig. 2.

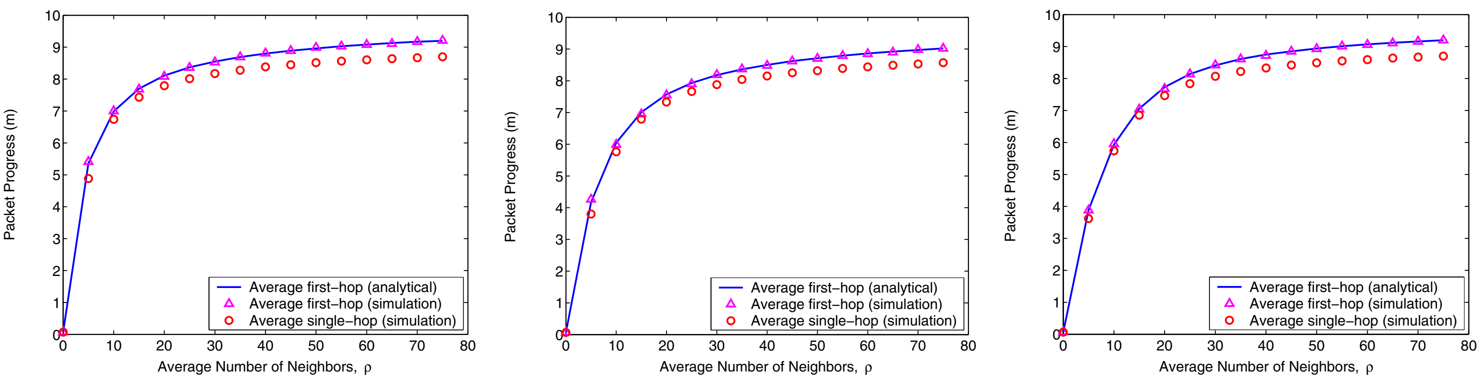
Our final but most important observation from Fig 6, as one of the conclusions from this work as well, is that, in terms of average single-hop packet progress, when ρ is small, i.e., from 0 to around 15 nodes, MFA performs the best, then MCA, followed by DRA; when ρ is medium, i.e., between 15 and 40 nodes, MFA still performs the best, DRA is the second and both outperform MCA; when ρ is large, i.e., 40 nodes up to infinity, DRA converges to the performance of MFA and both are better than MCA. It is expected that the performance of MCA will also converge to that of MFA as ρ goes up to a very large value (i.e., much larger than 100) or more. Note that DRA and MCA never out-perform MFA.

The above observation can be explained as follows: when ρ is small, the size of the forwarding area dominates the average single-hop packet progress since it determines the probability of void for that area; when ρ is large enough to counteract the effect of voids, the



(a) r = 10m and ρ = 5 nodes (b) r = 10m and ρ = 15 nodes (c) r = 10m and ρ = 50 nodes

Fig. 7. Numerical results for average single-hop packet progress vs network range



(a) MFA (b) MCA (c) DRA

Fig. 8. Simulations results for the validation of our analysis (r = 10m and x = 100m)

location of the forwarding area dominates the calculation of average single-hop packet progress since a forwarding area which covers more area that is farther away from the sender and is nearer to the destination will have better next-hop candidate nodes and thereby attain a larger average single-hop packet progress. Similar conclusions can be drawn from Fig. 7, which suggests that our observation holds for different values of network range.

The above results provide a theoretical basis for the design of a practical CGF protocol based on timer and receiver-overheard capability in a randomly distributed wireless ad hoc network. That is, without void handling, if ρ is known *a priori* to be small or medium, we should select MFA as the forwarding area; if ρ is 40 or more, DRA should be selected since it has the same network performance as MFA, and more importantly, a simpler MAC design and a less expensive radio capability. MCA should only be chosen when ρ is extremely high (i.e., much larger than 100).

In Fig. 7, we compare the average single-hop packet progress with respect to different values of network range x. Node transmission range is fixed at $10m$ while ρ is set to 5 nodes in Fig. 7(a), 15 nodes in Fig. 7(b), and 50 nodes in Fig. 7(c).

From Fig. 7, we observe that, MFA attains an average single-hop packet progress that is almost $1m$, i.e., 10%, higher than MCA and DRA, for $\rho = 5$ or $\rho = 15$. For $\rho = 50$, MFA and MCA achieve a $0.5m$, i.e., 5%, higher average single-hop packet progress than DRA. In the case of $\rho = 5$, when network range is small and is comparable to node transmission range, average single-hop packet progress plummets quickly and then maintains a relatively constant value as network range increases. In the case of $\rho = 15$ or $\rho = 50$, this trend is

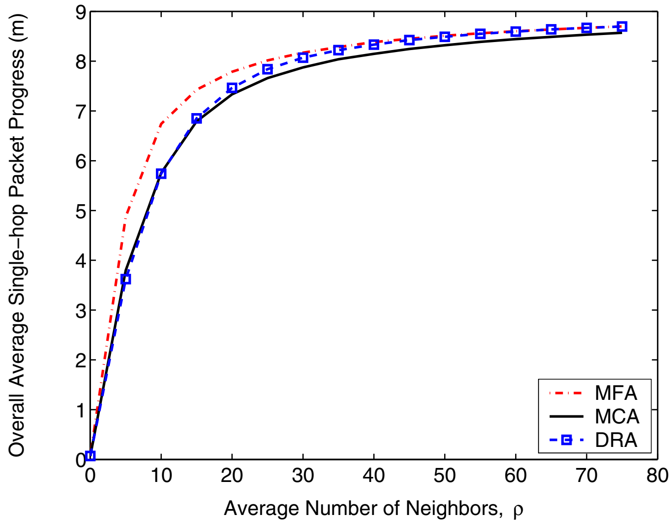


Fig. 9. Simulation results for the comparison of overall average single-hop packet progress (r = 10m and x = 100m)

reversed and average single-hop packet progress jumps to a higher level and stays at that level as x increases. This result is related to our 3-step forwarding strategy and can be explained as follows: when network range is small, only step (i) affects the result; however, when network range is much larger than the node transmission range, step (ii) and step (iii) also impact the result. When $\rho = 5$, step (iii) has a larger impact than step (ii), so the initial value is larger than the stable value. In the case of $\rho = 15$ or $\rho = 50$, step (ii) has a larger impact than step (iii) and the initial value is thereby lower than the stable value.

B. Simulation Results

Simulations (programs written in C language and run in Microsoft Visual C++ 6.0) have been performed to verify our analysis. In our simulations, the network nodes are distributed in a circular region according to a two-dimensional Poisson distribution. The circle is centered at $(0, 0)$ with radius 100 meters while the node transmission range is set to 10 meters. The source node is fixed at $(0, 0)$ while destination nodes are randomly chosen in the circle. The source node transmits packets to the selected destination node in accordance with our 3-step forwarding strategy while specifying a forwarding area as MFA, MCA, or DRA. We measured average first-hop packet progress, average single-hop packet progress (not only the first-hop, but also the average of all the hops from the source node to the destination) of each pair of source and destination. All the results presented in Figs. 8 and 9 are the average of 500 runs, each of which selects 100 destinations randomly.

As shown in Fig. 8, simulation results match well with our analytical results for average first-hop packet progress in MFA, MCA, and DRA. However, our actual desired performance measure, namely the overall average single-hop packet progress, is slightly lower than our analytically derived average first-hop packet progress, as expected. The main reason is that the last-hop packet progress, where a packet is directly delivered to a destination, is often less than the packet progress in the previous hops, where a packet is forwarded using a greedy approach.

Figure 9 confirms our observations made using analytical results. In fact, the effect of last-hop packet progress is similar for different forwarding areas. Extensive simulations have been performed and all the results showed the same tendency. Therefore, our results on first-hop packet progress can be employed as the overall single-hop packet progress to evaluate CGF with different forwarding areas.

V. The Effect of Void Handling Schemes

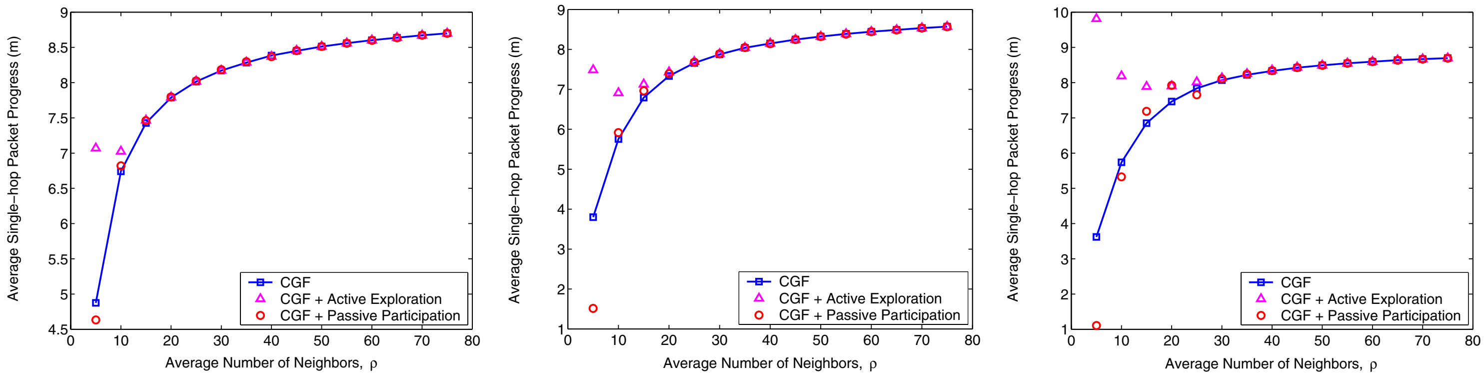
Thus far, we have studied the average single-hop packet progress performance of CGF without any void handling schemes for different forwarding areas. We further investigate the effect of void handling schemes on our previous results in this section.

A void occurs when a packet is at a node which has no immediate neighbors in the forwarding area. This may lead to the data packet being dropped by this node, even though the original source and the intended destination are topologically connected. A void handling scheme is necessary to somehow bypass the void.

Before a node initiates a void handling procedure, it should first determine whether the void is temporary due to packet collisions or temporary unavailability of next-hop nodes. Temporary voids will disappear as nodes wake-up, collisions resolved, and mobile nodes move in. Such temporary voids only affect the end-to-end packet delay.

Note that voids can be handled by state-based void handling schemes such as perimeter routing in GPSR [16] and BOUNDHOLE in [10], but these techniques require nodes to collect the local topology information either proactively or on demand. Thus, we focus on approaches that have the state-free feature by introducing two state-free void handling schemes for CGF here and study their performance via extensive simulations.

The first state-free void handling scheme is termed *active exploration*. In this scheme, when a node cannot



(a) MFA (b) MCA (c) DRA

Fig. 10. Simulation results for CGF with different forwarding areas (r = 10m and x = 100m)

locate a next-hop node in its forwarding area, it gradually increases its transmission range until a next-hop node is found in its updated forwarding area. Note that we still use location instead of topology information. Thus, it is a state-free scheme. The extra costs of this scheme are the additional delay and the requirement of increasing a node's transmission power (hence the transmission range). In this work, we assume that a node can increase its range as much as it needs to. This assumption may not be practical. In practice, however, if a node still does not find any next-hop candidate nodes after extending its transmission range to the maximum value, the following second scheme can be employed to avoid further loss of data packets.

The second state-free scheme is termed *passive participation*. The main idea is that once a node cannot locate a next-hop node, it simply discards the data packet and discourages itself from picking up any following data packets for that destination. The node may go back to check if there exist any next-hop nodes at a future time. This simple strategy has a reverse-propagation effect which eventually informs other intermediate next-hop nodes to avoid those nodes with voids for that destination. The disadvantage of this scheme is that it cannot be well exploited under low nodal density since most nodes and even the source node may be discouraged, which causes more serious network partition and more packet losses than before. Further analysis of these two schemes is not presented here due to page limitations.

The simulation experiment has the same configuration as in the last section and simulation results are shown in Figs. 10, 11, and 12. From Fig. 10, we can observe that MFA, MCA, and DRA have the same trend. That is,

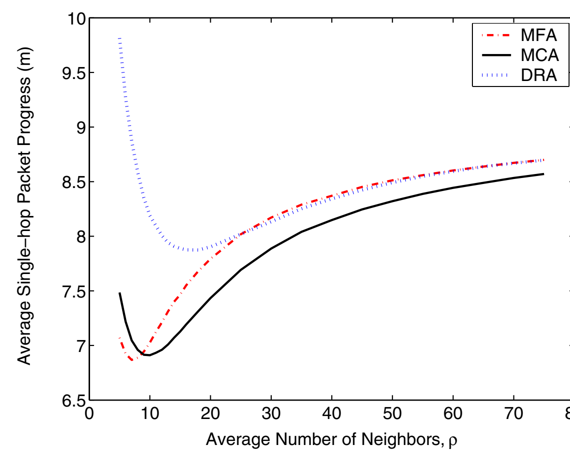


Fig. 11. Simulation results for CGF plus active exploration (r = 10m and x = 100m)

when ρ is small, CGF plus active exploration performs the best, then CGF without void handling, followed by CGF plus passive exploration. As ρ increases, the performance of the three cases begins to converge. However, the point of their convergence is different. For MFA, it is around 10 nodes; for MCA, it is around 15 nodes; while for DRA, it is around 25 nodes. This can be explained as follows: with a smaller ρ, the probability of void is larger, and in this case the void handling schemes will contribute more to average packet progress and vice versa. Thus, different probabilities of void with respect to ρ in these three schemes, as shown in Fig. 2, induce different convergence points.

Based on Fig. 11, we can see that active exploration is very effective for small values of ρ. However, we want to point out that this improvement is at the cost

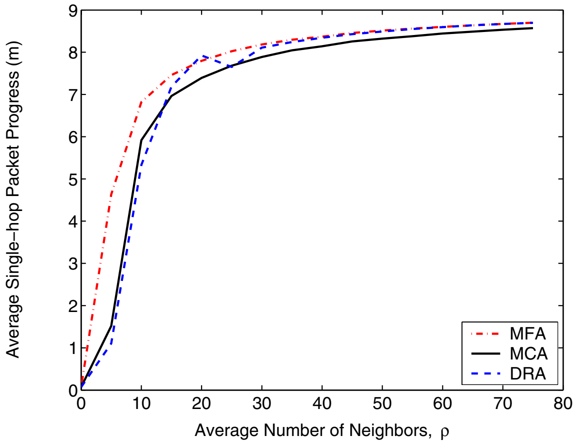


Fig. 12. Simulation results for CGF plus passive participation (r = 10m and x = 100m)

of extra energy consumption. In the extreme case, i.e., $\rho \approx 0$, CGF plus active exploration becomes a direct instead of multi-hop communication between the source and the destination, which has the largest average packet progress but also the most energy consumption.

In Fig. 12, we can see that CGF that uses passive participation is very ineffective and even worse than CGF without void handling for small values of ρ, since it can discourage most nodes and even cause more serious network partitioning. However, when ρ is moderate, it begins to function well. The advantage of this scheme is that it is simple, efficient, and easy to implement.

Note that although handling voids cannot improve average single-hop packet progress after ρ passes a threshold value, a state-free void handling scheme is still desirable for higher values of ρ, since it can improve packet delivery ratio and protect the network from any unexpected loss of important data at nodes with voids.

VI. Conclusions

In this paper, we have modeled Contention-based Geographic Forwarding (CGF) without void handling as a 3-step forwarding strategy and then constructed a general analytical framework. Our framework provides a theoretical basis for the selection of an effective forwarding area for the design of a practical CGF protocol, in terms of average single-hop packet progress. The framework, validated by numerical results and extensive simulations, can also serve as a performance evaluation technique for the CGF protocols that have been designed previously. Furthermore, we have also introduced two state-free void handling schemes and studied their performance when employed in conjunction with CGF via extensive simulations.

In our future work, it would be of interest to execute packet-level simulations to observe how our analysis matches with the results obtained from the existing CGF protocols. We also plan to obtain more analytical results in terms of other node distribution models as well as other performance metrics than those used in this work.

References

[1] H. Füssler, J. Widmer, M. Käsemann, M. Mauve, and H. Hartenstein, “Contention-Based Forwarding for Mobile Ad-Hoc Networks,” in *Ad Hoc Networks Journal (Elsevier)*, Vol. 1, No. 4, November 2003.

[2] M. Zorzi, R. R. Rao, “Geographic Random Forwarding (GeRaF) for Ad Hoc and Sensor Networks: Multihop Performance,” in *IEEE Transactions on Mobile Computing*, vol. 2, Oct.-Dec. 2003.

[3] B. M. Blum, T. He, S. Son, and J. A. Stankovic, “IGF: A Robust State-Free Communication Protocol for Sensor Networks,” in *Technical report CS-2003-11*, CS Department, University of Virginia, 2003.

[4] H. Füssler, H. hartenstein, J. Widmer, M. Mauve, and W. Effelsberg, “Contention-Based Forwarding for Street Scenarios,” in *proc. of the 1st international workshop on intelligent transportation (WIT2004)*, Hamburg, Germany, Mar 2004.

[5] D. Chen, J. Deng, and P. K. Varshney, “A State-Free Data Delivery Protocol for Wireless Sensor Networks,” in *proc. of IEEE WCNC 2005*, New Orleans, LA, USA, March 2005.

[6] A. Savvides, C. Han, and M. B. Strivastava, “Dynamic Fine-grained Localization in Ad-Hoc Networks of Sensors,” in *Proc. of ACM MobiCom 2001* , Rome, Italy, July 2001.

[7] N. Bulusu, J. Heidemann, and D. Estrin, “GPS-less Low Cost Outdoor Localization For Very Small Devices,” *IEEE Personal Communications Magazine*, 7(5), pp. 28-34, Oct. 2000.

[8] D. B. Johnson and D. A. Maltz, “Dynamic Source Routing in Ad Hoc Wireless Networks,” in *Mobile Computing*,1996.

[9] J. Li, J. Jannotti, D. S. J. De Couto, D. R. Karger, and R. Morris, “A Scalable Location Service for Geographical Ad Hoc Routing,” in *Proc. of ACM MobiCom 2000*, August 2000.

[10] Q. Fang, J. Gao, and L. J. Guibas, “Locating and Bypassing Routing Holes in Sensor Networks,” in *Proc. of IEEE Infocom 2004*, Hong Kong, March 2004.

[11] “IEEE Standard for Wireless LAN - Medium Access Control and Physical Layer Specification, P802.11 ,” 1999.

[12] J. L. Gao, “Analysis of Energy Consumption for Ad Hoc Wireless Sensor Networks Using a Bit-meter-per-joule Metric,” in *IPN Progress Report*, 42-150, August 2002.

[13] T. Melodia, D. Pompili, and Ian F. Akyildiz, “Optimal Local Topology Knowledge for Energy Efficient Geographical Routing in Sensor Networks,” in *proc. of IEEE INFOCOM 2004*, 42-150, Hong Kong, March 2004.

[14] J. Deng, Y. S. Han, P.-N. Chen, and P. K. Varshney, “Optimum Transmission Range for Wireless Ad Hoc Networks,” in *proc. of IEEE WCNC 2004*, Atlanta, GA, USA, March 2004.

[15] P. Billingsley, *Probability and Measure*, New York, NY: John Wiley and Sons, 1995.

[16] B. Karp and H. T. Kung, “Greedy Perimeter Stateless Routing for Wireless Networks,” in *Proc. of ACM MobiCom 2000*, Boston, MA, August 2000.