Loss Probability of LRD and SRD Traffic in Generalized Processor Sharing Systems

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Abstract

Generalized Processor Sharing (GPS) is an efficient and flexible scheduling mechanism for sharing server capacity and providing differentiated Quality-of-Service (QoS) owing to its appealing properties of fairness, traffic isolation, and work conservation. This paper analytically investigates the loss probabilities of individual traffic flows in GPS systems subject to heterogeneous Long-Range Dependent (LRD) and Short-Range Dependent (SRD) traffic, which have not been studied in the open literature. We derive and validate the closed-form expressions of the loss probabilities of both traffic flows. We then evaluate the effects of Hurst parameter of LRD traffic on the performance of GPS systems in terms of traffic loss probability.

1 Introduction

Traffic scheduling mechanisms are crucial to provide differentiated Quality-of-Service (QoS) (e.g., packet delay and loss) for a diverse spectrum of network applications. As a promising traffic scheduling mechanism, Generalized Processor Sharing (GPS) [13] has attracted tremendous research efforts owing to its appealing features, such as, fairness, traffic isolation, and work conservation [17, 18]. The GPS mechanism assigns each traffic flow a fixed weight which can then guarantee a minimum service rate for the flow even though other traffic flows may be greedy in demanding service. This property helps GPS achieve relative fairness among traffic flows, and meanwhile makes it possible to isolate different flows and provide differentiated QoS. GPS is work-conserving in that it can redistribute any excess service to backlogged traffic flows.

Long-Range Dependent (LRD) characteristics (i.e., large-lag correlation and scale-invariant burstiness) of network traffic have been discovered by many recent measurement studies [1, 7, 14]. The fractal-like nature of LRD traffic has significantly different theoretical properties from those of the conventional Short-Range Dependent (SRD) processes (e.g., Poisson processes, Markov modulated fluid processes [15]). However, most existing studies based on analytical models have been limited to GPS systems subject to either SRD traffic or LRD traffic only, such as, [5, 15, 19] for SRD traffic and [2, 3, 8, 18] for LRD traffic. Although packet loss probability, as one of the most important QoS metrics, plays a key role in the design, evaluation, and optimization of traffic scheduling mechanisms, no efforts have been reported in the open literature to analytically investigate the loss probability of GPS systems in the presence of heterogeneous LRD and SRD traffic.

Based on the analytical upper and lower bounds for the queue length distributions of GPS systems reported in [4], we derive the closed-form expressions of the loss probabilities of both flows in GPS systems fed with heterogeneous LRD fractional Brownian motion (fBm) traffic and SRD Poisson traffic. We demonstrate the validity and accuracy of the obtained loss probabilities through comparison between analytical results and extensive simulation results. Besides, we study the performance of GPS systems in terms of traffic loss probability under different working conditions. The results reveal that the loss probability of LRD traffic increases sharply as its Hurst parameter increases, but the loss probability of SRD traffic remains unchanged due to the flow isolation nature of GPS.

The rest of the paper is organized as follows. In Section 2, we introduce the characteristics and mathematical description of heterogeneous LRD fBm traffic and SRD Poisson traffic. We briefly review the upper and lower bounds for the tails of queue length distributions of individual traffic flows in GPS systems subject to heterogeneous fBm and Poisson traffic in Section 3. Next, Section 4 derives the loss probabilities of fBm and Poisson traffic. Section 5 validates the analytical results through extensive experimental simulations and analyzes the performance of GPS systems. Finally, Section 6 concludes the paper.

2 LRD versus SRD Traffic

This paper is intended to study the loss probabilities of heterogeneous traffic in GPS scheduling systems. Specifically, we address two types of traffic, namely, LRD fBm traffic and SRD Poisson traffic. In what follows, we will briefly review the modelling issues of LRD and SRD traffic.

Generally speaking, a traffic flow[1] can be modelled as a stochastic process and denoted in a cumulative arrival form as $\mathbf{A} = \{A(t)\}_{t\in\mathbb{N}}$, where $A(t)$ is the cumulative number of traffic arrivals at time t. Consequently, $A(s,t) = A(t) - A(s)$ denotes the amount of traffic arriving in time interval $(s,t]$. Traffic flow $\mathbf{A}$ can also be denoted in an increment form, i.e., $\mathbf{A} = \{B(t)\}_{t\in\mathbb{N}}$, where $B(t)$ is the traffic arriving in time interval $(t-1,t]$ with $B(0) = 0$. These two representation forms have the following relationship: $A(t) = \sum_{i=0}^{t} B(i)$ and $B(t) = A(t) - A(t-1)$.

Note that for the sake of clarity of the model derivation, hereafter we will use subscripts f and p to distinguish a given quantity of fBm and Poisson traffic, respectively.

2.1 LRD fBm traffic

Since the innovative study of Leland *et al.* [7] on LRD traffic, many models and techniques have been developed to characterize traffic long-range dependence or generate LRD traffic traces. Among the existing models, fractional Brownian motion (fBm) is identified as the most efficient and accurate way for modelling and generating LRD traffic [11]. An fBm traffic flow can be expressed as $\mathbf{A}_f = \{A_f(t)\}_{t\in\mathbb{N}}$ [11]:

$$A_f(t) = \lambda_f t + Z_f(t), \tag{1}$$

where λ_f is the mean arrival rate and $Z_f(t) = \sqrt{a_f\lambda_f}\bar{Z}_f(t)$. $\bar{Z}_f(t)$ is a centered (i.e., $E(\bar{Z}_f(t)) = 0$) fBm with variance $Var(\bar{Z}_f(t)) = t^{2H_f}$. The variance and covariance functions of $\mathbf{A}_f$ can be given as follows:

$$Var(A_f(t)) = a_f\lambda_f t^{2H_f}, \tag{2}$$

$$\Gamma_f(s,t) = \tfrac{1}{2}a_f\lambda_f\left(t^{2H_f} + s^{2H_f} - (t-s)^{2H_f}\right), \tag{3}$$

where $H \in (\frac{1}{2}, 1]$ is Hurst parameter indicating the degree of long-range dependence.

In the increment form, traffic flow $\mathbf{A}_f$ can be expressed as $\mathbf{A}_f = \{B_f(t)\}_{t\in\mathbb{N}}$ with mean arrival rate $E(B_f(t)) = \lambda_f$ and variance $Var(B_f(t)) = a_f\lambda_f$.

[1] All traffic flows are modelled in discrete time in this paper.

2.2 SRD Poisson traffic

Using the similar notation of fBm traffic in Eq. (1), a Poisson traffic flow can be denoted as $\mathbf{A}_p = \{A_p(t)\}_{t\in\mathbb{N}}$:

$$A_p(t) = \lambda_p t + Z_p(t), \tag{4}$$

where λ_p is the mean arrival rate of $A_p(t)$ and $Z_p(t)$ is a stochastic process with expectation $E(Z_p(t)) = 0$. The variance and covariance functions of $\mathbf{A}_p$ are as follows:

$$Var(A_p(t)) = \lambda_p t, \tag{5}$$

$$\Gamma_p(s,t) = \lambda_p \min(s,t). \tag{6}$$

Similarly, the Poisson traffic flow can be expressed in the increment form as $\mathbf{A}_p = \{B_p(t)\}_{t\in\mathbb{N}}$ with mean arrival rate $E(B_p(t)) = \lambda_p$ and variance $Var(B_f(t)) = \lambda_p$.

3 Upper and Lower Bounds of the Tails of Queue Length Distributions

For the sake of the model description, we define some notations first. Let C be service capacity of the GPS system, and μ_f and μ_p represent the weights assigned to fBm and Poisson traffic flows, respectively. The guaranteed service rates for fBm and Poisson traffic can then be denoted as $\mu_f C$ and $\mu_p C$. $Q_f(t)$ and $Q_p(t)$ denote queue lengths of fBm and Poisson traffic, respectively, at time t.

In [9, 10], an approach based on large deviation principle was developed to derive the upper and lower bounds of the aggregate queue length distribution of GPS systems subject to general Gaussian traffic. By approximating a Poisson traffic flow as a Gaussian process, the above approach is extended to deal with heterogeneous LRD fBm traffic and SRD Poisson traffic in [4]. More specifically, the analytical upper and lower bounds for the tails (i.e., $\mathbb{P}(Q > x)$) of the queue length distributions of individual traffic flows were developed.

In order to address their queue length distributions, individual traffic flows are classified as *Primary Queue Contributors (PQCs)* or *Secondary Queue Contributors (SQCs)* of the GPS system in [4]. PQCs refer to traffic flows with high arrival rates, large variances of arrivals, and/or low guaranteed service rates, whilst SQCs are traffic flows with low arrival rates, small variances of arrivals, and high guaranteed service rates. Obviously, PQCs cannot be served timely and thus act as the dominating contributors of the aggregate queue. On the other hand, SQCs make minor contribution to the aggregate queue because their arrivals can be handled in time. Therefore, in a two-queue GPS system the queue length distribution of the PQC traffic flow can be reasonably approximated by that of the aggregate queue. Further, the upper and lower bounds for the tail of the queue length distribution of the PQC traffic flow can be given as follows [4]:

$$\mathbb{P}(Q_{pqc} > x) \leq \exp\left(-\frac{1}{2}Y(t_x)\right), \tag{7}$$

$$\mathbb{P}(Q_{pqc} > x) \geq \bar{\Phi}\left(|\sqrt{Y(t_x)}|\right), \tag{8}$$

where

$$Y(t) = \frac{(-x + (C - \lambda_f - \lambda_p)t)^2}{\Gamma_f(t,t) + \Gamma_p(t,t)}, \tag{9}$$

and $t_x = \arg\min_t Y(t)$. $\bar{\Phi}(\cdot)$ is the residual distribution function of the standard Gaussian distribution. A commonly adopted approximation is $\bar{\Phi}(x) \approx \exp(-\frac{1}{2}x^2)/\sqrt{2\pi(1+x)^2}$.

Since an SQC makes minor contribution to the aggregate queue of a GPS system, it seems to be served in a manner as if its arrivals are handled in an isolated system with its guaranteed service rate as the service capacity of the system. Therefore, the upper and lower bounds for the tail of the queue length distribution of the SQC traffic flow can be derived as follows [4]:

$$\mathbb{P}(Q_{sqc} > x) \leq \exp\left(-\frac{1}{2}Y_i(t_x)\right), \tag{10}$$

$$\mathbb{P}(Q_{sqc} > x) \geq \bar{\Phi}\left(Y_i(t_x)\right), \tag{11}$$

where i represents f if fBm traffic is SQC, otherwise p for Poisson traffic.

$$Y_i(t) = \frac{(-x + (\mu_i C - \lambda_i)t)^2}{\Gamma_i(t,t)}, \tag{12}$$

and $t_x = \arg\min_t Y_i(t)$.

It has been experimentally proven in [4] that although the derivation of the bounds of individual queue length distributions is subject to the distinction of the PQC and SQC traffic flows, the developed bounds are applicable to the case where the difference between the contribution of fBm and Poisson traffic flows to the aggregate queue length is relatively small.

4 Loss Probabilities of Individual Traffic Flows

In this section, we estimate the loss probabilities, $\mathcal{P}_L(x)$, of both fBm and Poisson traffic flows, respectively, in the GPS system based on the tails (i.e., $\mathbb{P}(Q > x)$) of individual queue length distributions presented in Section 3. Given that the GPS system is stable (i.e., $\lambda_f + \lambda_p < C$), the relationship between loss probability and the tail of queue length distribution can be given as follows [6]:

$$\frac{\mathcal{P}_L(x)}{\mathbb{P}(Q > x)} = \frac{\mathcal{P}_L(b)}{\mathbb{P}(Q > b)}, \tag{13}$$

where b is an arbitrary constant. Let $\alpha = \mathcal{P}_L(b)/\mathbb{P}(Q > b)$. Eq. (13) can be rewritten as

$$\mathcal{P}_L(x) = \alpha\mathbb{P}(Q > x). \tag{14}$$

In what follows, we will present an approximation to $\mathbb{P}(Q > x)$ and calculate α for both PQC and SQC traffic flows in order to obtain their loss probabilities.

Given $\bar{\Phi}(x) \approx \exp(-\frac{1}{2}x^2)/\sqrt{2\pi(1+x)^2}$, the difference between the upper and lower bounds for the tail of the queue length distribution of the PQC traffic flow is the coefficients of $\exp(-\frac{1}{2}Y(t_x))$. Examining the upper and lower bounds corresponding to the SQC traffic flow reveals the same phenomenon. This finding inspires us to take a certain mean (e.g., arithmetic, geometric) of the upper and lower bounds for the tail of a queue length distribution as its approximation. In this paper, we employ the geometric mean that has been proven effective in [16]. As a result, we have the following approximations to the PQC and SQC traffic flows, respectively:

$$\mathbb{P}(Q_{pqc} > x) \approx \frac{1}{\sqrt[4]{2\pi(1+Y(t_x))^2}} \exp(-\frac{1}{2}Y(t_x)), \tag{15}$$

$$\mathbb{P}(Q_{sqc} > x) \approx \frac{1}{\sqrt[4]{2\pi(1+Y_i(t_x))^2}} \exp(-\frac{1}{2}Y_i(t_x)). \tag{16}$$

Next, we calculate α_{pqc} and α_{sqc} for the PQC and SQC traffic flows, respectively. For a Gaussian traffic flow $\mathbf{A} = \{B(t)\}_{t\in\mathbb{N}}$ with mean arrival rate $E(B(t)) = \lambda$ and variance $Var(B(t)) = \sigma^2$, if setting $b = 0$ the constant α can be readily calculated [6]. Let c be the service rate obtained by the Gaussian traffic flow. α can then be calculated as follows:

$$\alpha = \frac{1}{\lambda\sqrt{2\pi}\sigma} \exp\left(\frac{(c-\lambda)^2}{2\sigma^2}\right) \times \int_c^{\infty} (z-c) \exp\left(-\frac{(z-\lambda)^2}{2\sigma^2}\right) dz. \tag{17}$$

- α_{pqc} for the PQC traffic flow

  In [4], the Poisson traffic flow is approximated as a Gaussian one. Actually, the feasibility of such an approximation has been widely proven. When the mean arrival rate of the Poisson traffic flow is large or the process time tends to infinity, the approximation is considerably exact. Therefore, in this paper we also approximate the Poisson traffic flow $\mathbf{A}_p$ as a Gaussian one with mean arrival rate $E(B_f(t)) = \lambda_p$ and variance $Var(B_f(t)) = \lambda_p$. As a result, the aggregate traffic flow, $\mathbf{A}_{\{f,p\}}$, of fBm and Poisson traffic can be regarded as a Gaussian process, which has mean arrival rate

$$E(B_{\{f,p\}}(t)) = E(B_f(t) + B_p(t)) = \lambda_f + \lambda_p, \tag{18}$$

and variance

$$Var(B_{\{f,p\}}(t)) = Var(B_f(t)+B_p(t)) = a_f\lambda_f + \lambda_p. \quad (19)$$

Obviously, the service rate obtained by the aggregate traffic flow is equal to the service capacity, C, of the GPS system. Therefore, by substituting $c = C$, $\lambda = \lambda_f + \lambda_p$ (Eq. (18)), and $\sigma = \sqrt{a_f\lambda_f + \lambda_p}$ (Eq. (19)) into Eq. (17), we obtain α_{pqc}:

$$\alpha_{pqc} = \frac{1}{(\lambda_f+\lambda_p)\sqrt{2\pi(a_f\lambda_f+\lambda_p)}} \times \exp\left(\frac{(C-(\lambda_f+\lambda_p))^2}{2(a_f\lambda_f+\lambda_p)}\right) \times \int_C^{\infty}(z-C)\exp\left(-\frac{(z-(\lambda_f+\lambda_p))^2}{2(a_f\lambda_f+\lambda_p)}\right)dz. \quad (20)$$

- α_{sqc} for the SQC traffic flow

  To obtain α_{sqc}, we need to address the following two cases. If fBm traffic acts as the SQC, upon substitution of $c = \mu_f C$, $\lambda = \lambda_f$, and $\sigma = \sqrt{a_f\lambda_f}$ into Eq. (17), we obtain α_{sqc} as follows:

$$\alpha_{sqc} = \frac{1}{\lambda_f\sqrt{2\pi a_f\lambda_f}}\exp\left(\frac{(\mu_f C-\lambda_f)^2}{2a_f\lambda_f}\right) \times \int_{\mu_f C}^{\infty}(z-\mu_f C)\exp\left(-\frac{(z-\lambda_f)^2}{2a_f\lambda_f}\right)dz. \quad (21)$$

  If Poisson traffic acts as the SQC, following the above Gaussian approximation of the Poisson traffic flow, we may substitute $c = \mu_p C$, $\lambda = \lambda_p$, and $\sigma = \sqrt{\lambda_p}$ into Eq. (17) to obtain α_{sqc} as follows:

$$\alpha_{sqc} = \frac{1}{\lambda_p\sqrt{2\pi\lambda_p}}\exp\left(\frac{(\mu_p C-\lambda_p)^2}{2\lambda_p}\right) \times \int_{\mu_p C}^{\infty}(z-\mu_p C)\exp\left(-\frac{(z-\lambda_p)^2}{2\lambda_p}\right)dz. \quad (22)$$

Upon obtaining α_{pqc} and α_{sqc} and following Eq. (14), the loss probabilities of individual fBm and Poisson traffic flows can be derived by integrating Eqs. (15) and (20), and Eqs. (16) and (21)/(22), respectively.

5 Model Validation and Analysis

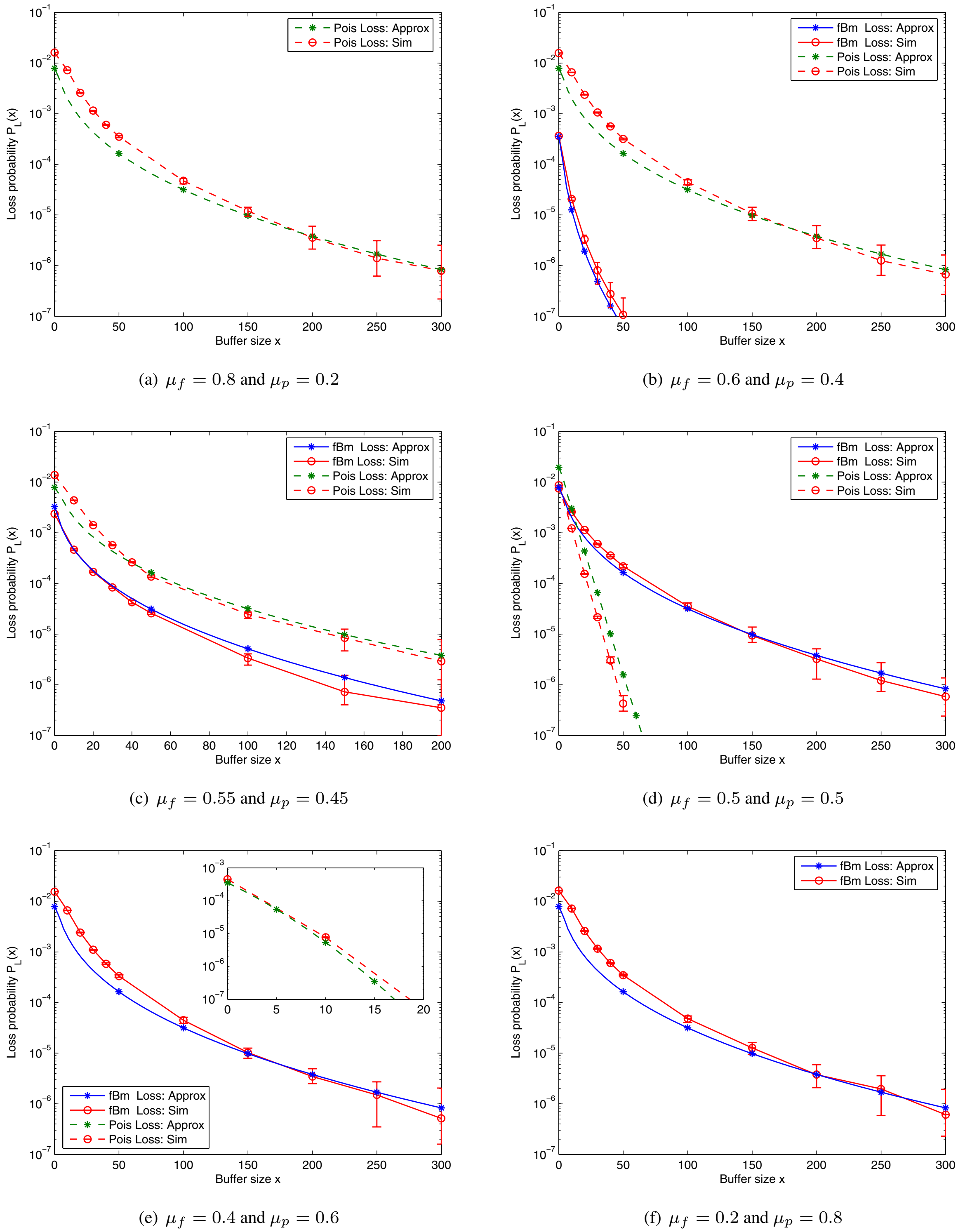
This section investigates the accuracy and analyzes the performance of the derived analytical loss probability model of individual traffic flows based on comparison between analytical and simulation results. To facilitate the investigation, we developed a simulator for the GPS system with the C++ programming language. In our simulation, the conditionalized Random Midpoint Displacement algorithm (RMD$_{3,3}$) [12] was adopted to generate fBm traffic traces because the computational complexity of this algorithm is linear with respect to the simulation trace length. Besides, we employed the method of batch mean to calculate 95% confidence intervals for all loss probabilities obtained through simulations.

5.1 Model Validation

In order to examine the validity and accuracy of the analytical model, we have conducted extensive simulation experiments under various scenarios corresponding to different parameter settings. We have reached consistent performance conclusions when different scenarios were taken into account. To specifically illustrate the accuracy of the derived model, in what follows we will present the performance results of a typical scenario, where the parameters are set as follows: server capacity $C = 120$, Hurst parameter $H_f = 0.8$, variance coefficient $a_f = 1.0$, and mean arrival rates $\lambda_f = 55$ and $\lambda_p = 55$. Under this scenario, we further studied different combinations of weights, μ_f and μ_p, of fBm and Poisson traffic flows so as to examine the effect of different weight combinations on the accuracy of the model. Figure 1 shows the detailed results of this scenario.

First of all, Figure 1 reveals that the simulation results of fBm (i.e., the solid curves with sign '$\circ$') and Poisson (i.e., the dashed curves with sign '$\circ$') traffic flows closely match the corresponding analytical estimations (i.e., the solid curve with sign '$*$' for fBm traffic and the dashed curve with sign '$*$' for Poisson traffic). This finding highlights the fact that the analytical model performs fairly well.

It is worth noting that in the cases shown in Figures 1 (a)-(c), fBm traffic acts as the SQC and can be served timely whilst Poisson traffic, acting as the PQC, cannot be handled in time because fBm traffic is assigned larger guaranteed service rates than Poisson traffic (i.e., $\mu_f > \mu_p$). As a result, Eqs. (16) and (21) are adopted to produce the analytical loss probability of fBm traffic in Figures 1 (a)-(c), while the corresponding curves for Poisson traffic are plotted using Eqs. (15) and (20). On the contrary, fBm traffic acts as the PQC while Poisson traffic as the SQC in the cases shown in Figures 1 (d)-(f). Therefore, Eqs. (15) and (20) are used to depict the analytical loss probability curves corresponding to fBm traffic; Eqs. (16) and (22) for Poisson traffic, respectively. Figure 1 (d) reveals a special case where fBm and Poisson traffic flows are assigned the same weights ($\mu_f = \mu_p = 0.5$) and their mean arrival rates are also identical ($\lambda_f = \lambda_p = 55$). However, due to the bursty feature over multiple time scales, fBm traffic is more likely to be piled up in its buffer than Poisson traffic. Thus, fBm traffic acts as the PQC in Figure 1 (d).



(a) $\mu_f = 0.8$ and $\mu_p = 0.2$

(b) $\mu_f = 0.6$ and $\mu_p = 0.4$

(c) $\mu_f = 0.55$ and $\mu_p = 0.45$

(d) $\mu_f = 0.5$ and $\mu_p = 0.5$

(e) $\mu_f = 0.4$ and $\mu_p = 0.6$

(f) $\mu_f = 0.2$ and $\mu_p = 0.8$

Figure 1. Comparison between the analytical and simulation results of loss probabilities in a typical scenario: server capacity $C = 120$, Hurst parameter $H_f = 0.8$, variance coefficient $a_f = 1.0$, and mean arrival rates $\lambda_f = 55$ and $\lambda_p = 55$.

An important phenomenon in Figures 1 (a) and (f) is that only the curves representing the analytical and simulation results of the PQC traffic flow, rather then SQC flow, can be plotted. This is due to the reason that the mean arrival rates of the SQC traffic flows in these two cases are much smaller than their guaranteed service rates. Consequently, the buffers of these SQC traffic flows keep empty and no packets are lost.

It is interesting to see that Figure 1 (c) reveals an important fact: The curves corresponding to simulation results of Poisson and fBm traffic, respectively, are close to each other. That is to say, although Poisson and fBm traffic flows act as the PQC and SQC, respectively, in the case with $\mu_f = 0.55$ and $\mu_p = 0.45$, the difference between their contribution to the aggregate queue is considerably small. However, even in this special case, the derived analytical loss probabilities for the PQC and SQC traffic flows are still accurate.

5.2 Performance Analysis

In what follows, we will examine the effect of LRD fBm traffic on the performance of GPS in terms of loss probability. To this end, we examined three values of Hurst parameter, namely, $H_f = \{0.7, 0.8, 0.9\}$ in a typical scenario with server capacity $C = 120$, variance coefficient $a_f = 1.0$, mean arrival rates $\lambda_f = 70$ and $\lambda_p = 30$, and weights $\mu_f = 0.6$ and $\mu_p = 0.4$. This is a typical setting in that fBm traffic has a relatively large mean arrival rate and is assigned a large weight, while these two parameters of Poisson traffic are relatively smaller. As a result, fBm traffic acts as the PQC, while Poisson traffic is the SQC. The results are shown in Figure 2.

It can be found from Figure 2 that as Hurst parameter H_f increases from 0.7 to 0.9, the loss probability of fBm traffic increases sharply. This phenomenon can be explained as follows. The larger Hurst parameter H_f is, the higher the probability that bursts of fBm traffic are followed by each other. As a result, such traffic burstiness spanning over many time scales gives rise to extended periods of large queue build-ups and consequently more packets of fBm traffic are lost due to buffer overflow.

Despite of the increase of Hurst parameter H_f from 0.7 to 0.9, the loss probabilities of Poisson traffic remains unchanged (see Figure 2). This is because Poisson traffic is always guaranteed a service rate larger than its mean arrival rate. Therefore, it is not affected by the changes of fBm traffic. This is exactly due to the flow isolation property of the GPS scheduling mechanism.

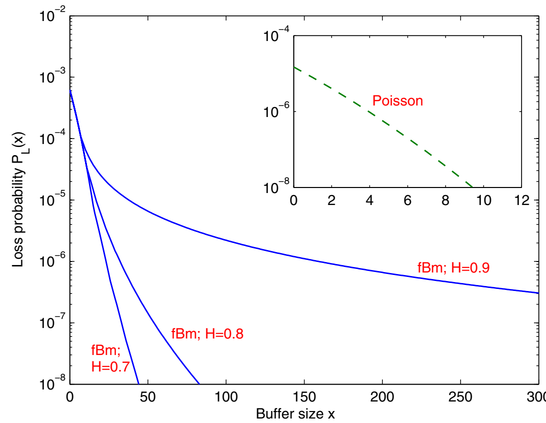


Figure 2. The effect of Hurst parameter H_f of fBm traffic on loss probabilities.

6 Conclusions

Traffic loss probability is a primary QoS metric and plays an important role in the design and performance of scheduling systems. Many recent studies have convincingly revealed the noticeable LRD nature of network traffic. This paper has investigated the loss probability of heterogeneous LRD fBm traffic and SRD Poisson traffic in GPS systems. We first presented an approximation to the tails of queue length distributions of individual traffic flows. We then derived the closed-form formula for their loss probabilities. Through comparisons between analytical and extensive simulation results, we demonstrated that the derived analytical model possesses a good degree of accuracy in predicting the loss probabilities of individual traffic flows under various working conditions. We also studied the performance of GPS under different settings of Hurst parameter of LRD fBm traffic. We found that the larger Hurst parameter, the higher the loss probability of LRD fBm traffic. However, because of the flow isolation property of the GPS scheduling mechanism, the loss probability of Poisson traffic keeps unchanged as Hurst parameter varies.

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