A COMBINED BAYESSHRINK WAVELET-RIDGELET TECHNIQUE FOR IMAGE DENOISING

Nezamoddin N. Kachouie and Paul Fieguth

University of Waterloo, Department of Systems Design Engineering, Waterloo, Canada

ABSTRACT

In this paper a combined BayesShrink Wavelet-Ridgelet denoising method is presented. In our previous work we have showed that BayesShrink Ridgelet performs better than VisuShrink Ridgelet and VisuShrink Wavelet. Although our BayesShrink Ridgelet technique performs somewhat poorer in comparison with BayesShrink Wavelet, based on SNR, visually it produces smoother results, especially for images with straight lines. In the proposed method BayesShrink Wavelet is combined with BayesShrink Ridgelet denoising method which performs better than each filter individually. The proposed combined denoising method gains the advantage of each filter in its specific domain, i.e., Wavelet for natural and Ridgelet for straight regions, and produces better and smoother results, both visually and in terms of SNR.

1. INTRODUCTION

Data obtained from the real world in the form of signals do not exist without noise. This noise might decrease to some negligible levels under ideal conditions such that denoising is not necessary, but usually to recover the signal the corrupting noise must be removed for practical purposes. For this reason noise elimination is a significant concern in computer vision and image processing. Noise undesirably corrupts the image by perturbations which are not related to the scene under study and makes ambiguities in the underlying signal relative to its observed form. The goal of denoising is to remove the noise and to retain the important signal features as much as possible. In the presence of additive noise, linear filters, which consist of convolving the image with a constant matrix to obtain a linear combination of neighborhood values, can produce a blurred and smoothed image with poor feature localization and incomplete noise suppression. To overcome these shortcomings, nonlinear filters have been proposed, especially wavelet based denoising [1, 2]. The wavelet transform generally separates signal and noise, as a result it can be used to remove the noise while preserving the signal characteristics. Researchers have employed various approaches to nonlinear wavelet-based denoising: wavelet thresholding, wavelet shrinkage, and others.

Fig. 1. Noisy image, SNR=7.93

To compensate the weaknesses of the wavelet transform to represent 1-D singularities in two-dimensional (2-D) signals, Ridgelet and Curvelet transforms were recently introduced by Candes and Donoho [4, 5]. The VisuShrink Ridgelet thresholding method was then introduced [6] as an alternative to the VisuShrink Wavelet denoising and performed better than its Wavelet counterpart for images with straight lines. Similar BayesShrink methods have been also introduced by [3, 7]. It seems quite reasonable to take advantage of the Ridgelet transform for improving the performance of the Wavelet transform in straight regions. Hence, a combined denoising method is proposed in this paper. To improve the visual image quality, the proposed method gains the advantage of Ridgelet transform in straight regions while the better performance of the Wavelet transform for natural images is considered. The paper presents an overview of the Ridgelet transform and BayesShrink denoising, followed by the proposed combined BayesShrink Wavelet Ridgelet image denoising method and experimental results.

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2. BACKGROUND OVERVIEW

The BayesShrink Wavelet method was introduced by Chang et al [3, 8] and is the state of the art in wavelet image denoising, outperforming the previous VisuShrink Wavelet method. We are interested in combining BayesShrink with the recently introduced Ridgelet transform [4, 5, 9, 10]. In this section an overview of BayesShrink and the Ridgelet transform are presented.

2.1. Ridgelet Transform

The Ridgelet transform effectively represents line singularities of 2-D signals. Since a sparse representation of smooth functions and straight edges is provided by the ridgelet transform, this new expansion can accurately represent both smooth functions and edges with fewer nonzero coefficients and achieves a lower mean square error (MSE) than the wavelet transform. It maps the line singularities into point singularities in the Radon domain by employing the embedded Radon transform. Therefore, the wavelet transform can efficiently be applied to discover the point singularities in this new domain. Having the ability to approximate singularities along a line, several terms with common ridge lines can effectively be superposed by the ridgelet transform. The bivariate ridgelet transform in R^2 is defined by

$$\Re_{\alpha,\beta,\theta}(\kappa) = \alpha^{-1/2}\omega((\kappa_1 \cos\theta + \kappa_2 \sin\theta - \beta)/\alpha) \quad (1)$$

where, $\alpha > 0, \beta$ and θ are scale, location and orientation parameters respectively and ω is a univariate wavelet function on $R \rightarrow R$. Along the Ridgelet lines $\kappa_1 \cos\theta + \kappa_2 \sin\theta$, Ridgelets are constant and they are equal to the wavelets in the orthogonal direction. Ridgelet coefficients of a bivariate function $I(\kappa)$ in R^2 are given by

$$\Re_I(\alpha, \beta, \theta) = \int \Re_{\alpha,\beta,\theta}(\kappa) I(\kappa) d\kappa \quad (2)$$

The reconstruction formula is given by

$$I(\kappa) = \int_0^{2\pi} \int_{-\infty}^{+\infty} \int_0^{\infty} \Re_I(\alpha, \beta, \theta) \Re_{\alpha,\beta,\theta}(\kappa) \frac{d\alpha}{\alpha^3} d\beta \frac{d\theta}{4\pi}$$

and is valid for integrable (and square integrable) functions. Like Fourier and Wavelet transforms, any arbitrary function can be represented by a continuous superposition of Ridgelets. Considering the 2-D Ridgelet transform as a 1-D Wavelet transform in the Radon domain, the Ridgelet coefficients of function $I(\kappa)$ can be defined as

$$\Re_I(\alpha, \beta, \theta) = \int \Re_t(\theta, \tau) \alpha^{-1/2} \omega((\tau - \beta)\alpha) d\tau \quad (3)$$

where $\Re_t(\theta, \tau)$ is the Radon transform of function $I(\kappa)$ and is based on the Dirac distribution (δ) as

$$\Re_t(\theta, \tau) = \int I(\kappa_1, \kappa_2) \delta(\kappa_1 \cos\theta + \kappa_2 \sin\theta - \tau) d\kappa_1 d\kappa_2$$

		Each Col Correspond to A Specific Direction N Col representing N Directions 1-D Subband Detail Coef. Level 1	
		1-D Subband Detail Coef. Level 2 Detail 2	
		. .	
		1-D Subband Detail Coef. Level L	
		Residual (Approximate) Coef. Level L	

Table 1. Ridgelet Coefficients

2.2. BayesShrink Method

The subband Wavelet and Ridgelet coefficients of a natural image can be described by the Generalized Gaussian Distribution (GGD) [3, 8, 7]:

$$GG_{\sigma,I,\gamma}(I) = P(\sigma_I, \gamma) \exp\{-[\delta(\sigma_I, \gamma)|I|]^{\gamma}\} \quad (4)$$

where $-\infty < I < +\infty, \gamma > 0$ and,

$$\delta(\sigma_I, \gamma) = \sigma_I^{-1} \left[\frac{\Gamma(3/\gamma)}{\Gamma(1/\gamma)}\right]^{\frac{1}{2}}$$

and,

$$P(\sigma_I, \gamma) = \frac{\gamma \cdot \delta(\sigma_I, \gamma)}{2\Gamma(\frac{1}{\gamma})}$$

σ_I is the standard deviation of subband Wavelet (or subband Ridgelet) coefficients, γ is the shape parameter and Γ is Gamma function. For the most natural images the distribution of the Wavelet (and Ridgelet) coefficients in a subband can be described with a shape parameter in the range of $[0.5, 1]$. Considering such a distribution for the Wavelet (or Ridgelet) coefficients and estimating and γ and σ_I for each subband, the soft threshold T_S which minimizes the Bayesian Risk [3, 8], can be obtained by

$$\Re(T_S) = E(\hat{I} - I)^2 = E_I E_{J|I} (\hat{I} - I)^2 \quad (5)$$

where $\hat{I}$ is $T_S(J)$, $J|I$ is $N(I, \sigma)$ and I is $GG_{\sigma_I,\gamma}$. Then the optimal threshold T_S^* is given by

$$T_S^*(\sigma_I, \gamma) = arg \min_{T_S} \Re(T_S) \quad (6)$$

It does not have a closed form solution and numerical calculation is used to find T_S^*. A proper estimation of the value T_S^* is concluded by setting the threshold as

$$\hat{T}(\hat{\sigma_I}) = \frac{\hat{\sigma_n}}{\hat{\sigma_n}} \quad (7)$$

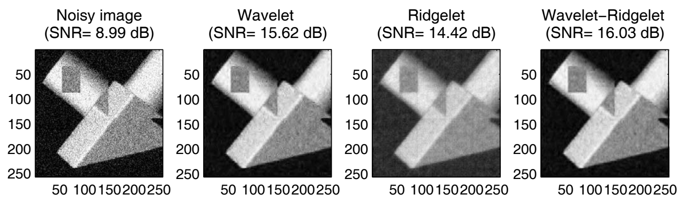

Fig. 2. BayesShrink Methods to Restore Noisy image

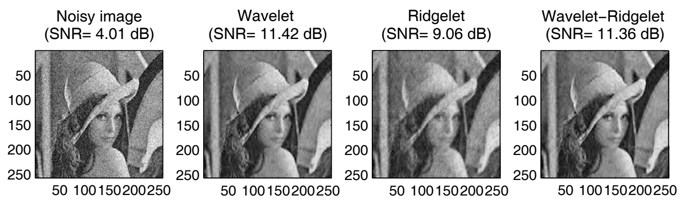

Fig. 3. BayesShrink Methods to Restore Noisy Lena Image

3. THE PROPOSED METHOD

The visual image quality of BayesShrink Ridgelet for straight regions is better than its Wavelet counterpart. At the same time BayesShrink Wavelet performs better on natural images. The proposed method by combining them, performs better than each method individually. The proposed denoising method is presented in this section.

3.1. Calculating the BayesShrink threshold

Subband dependent threshold is used to calculate BayesShrink Ridgelet threshold. The estimated threshold is given by [7] where n and I are noise and signal standard deviations respectively. The 1-D Ridgelet coefficients corresponding to different directions are depicted in Tab. 1. In this figure each column corresponds to a specific direction, hence the number of columns determines the number of directions and each column contains subband detail coefficients for L different decomposition levels. To estimate the noise variance σ_n^2 from the subband details, the median estimator is used on the 1-D subband coefficients:

$$\hat{\sigma_n} = median(|Details|)/0.6745 \tag{8}$$

Signal standard deviation is calculated for each direction in each subband detail individually. Thus having N directions and L subband, NxL different σ_I must be estimated corresponding to NxL subband-directions coefficients. Note that in BayesShrink Wavelet denoising, σ_I is estimated on 2-D dyadic subbands [3]. Thus having L decomposition levels, $3 \cdot L$ different σ_I must be estimated to calculate the thresholds for the different subbands. To estimate the signal standard deviation (σ_I), the observed signal S is considered to be $S = I + n$, while signal (I) and noise (n) are assumed to be independent. Therefore, $\sigma_S^2 = \sigma_I^2 + \sigma_n^2$ where σ_S^2 is the variance of the observed signal. So $\hat{\sigma_I}$ is estimated by

$$\hat{\sigma_I} = \sqrt{max((\hat{\sigma}_S^2 - \hat{\sigma}_n^2), 0)} \tag{9}$$

3.2. Combined Denoising Algorithm

To avoid the overhead complexity that signal synthesizing methods such as Basis Pursuit and Matching Pursuit cause, the observed image passes through Wavelet and Ridgelet denoising filters sequentially in the proposed method. Assuming $S = I + n$, the noise variance is estimated and the denoising task will be repeated till $|I - \hat{S}|^2 < e$ be satisfied, where I and S are observed and noise free signals respectively and e is a fraction of noise variance.

$e = \alpha\hat{\sigma_n^2}$
$\hat{S} = I$
do

$$\begin{aligned} &\{ \\ C_w &= \text{Wavelet Transform}\{\hat{S}\} \\ C_{T_w} &= \text{BayesShrink Wavelet}\{C_w\} \\ \hat{S} &= \text{Inverse Wavelet Transform}\{C_{T_w}\} \\ C_r &= \text{Ridgelet Transform}\{\hat{S}\} \\ C_{T_r} &= \text{BayesShrink Ridgelet}\{C_r\} \\ \hat{S} &= \text{Inverse Ridgelet Transform}\{C_{T_r}\} \\ &\} \end{aligned}$$

while($|I - \hat{S}|^2 > e$)

4. RESULTS

Some results obtained by applying BayesShrink Wavelet, BayesShrink Ridgelet and the proposed method are presented in this section. A synthetic image with straight lines is used in the first experiment. The restored images using three BayesShrink methods are depicted in Fig. 2. As we can observe, the combined filtering performed better than the two others such that its results have better visual quality and higher SNR. Fig. 3 shows the denoised Lena image using BayesShrink Wavelet, BayesShrink Ridgelet and the proposed method. Although this image does'nt have much straight regions, the combined BayesShrink Wavelet-Ridgelet does not degrade the superior result of Wavelet in this domain and performs about the same as BayesShrink Wavelet based on SNR while the combined method produces smoother result that can be observed by a closer look. At the end the proposed method is applied to Gold-hill which is a natural image with straight regions. Fig. 1 shows the noisy image and restored images using BayesShrink Wavelet and the proposed method are depicted in Fig. 4 and

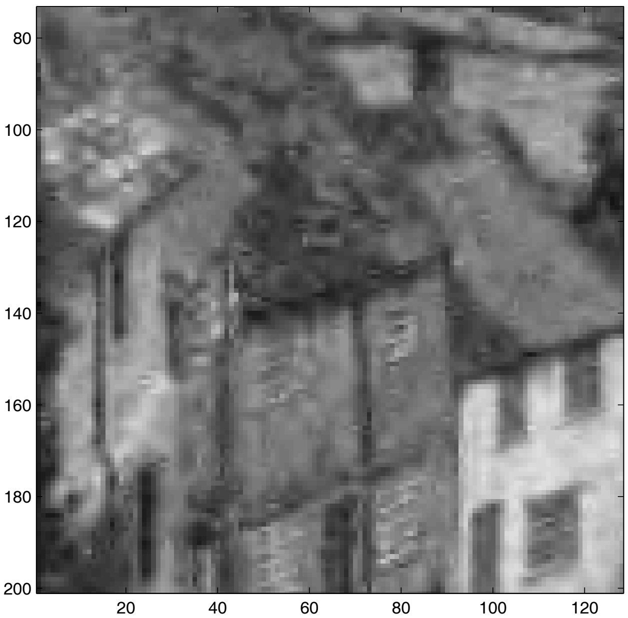

Fig. 4. BayesShrink Wavelet Denoising, SNR = 11.59

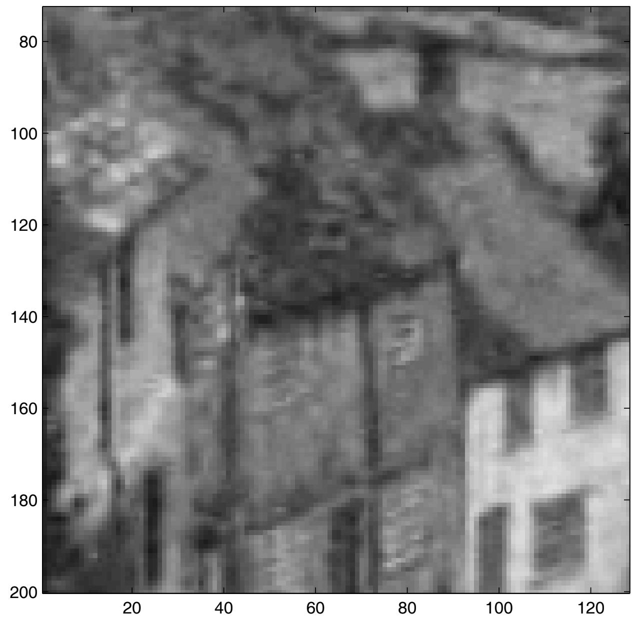

Fig. 5. BayesShrink Wavelet-Ridgelet, SNR = 11.82

5 respectively. The results are zoomed in for better observation. As it can be observed not only the SNR is improved by the combined method but also the image looks smoother, especially in the straight regions.

5. CONCLUSIONS

In this paper a combined denoising method was proposed. By addressing our previous work using Ridgelet transform for image denoising, a combined BayesShrink Wavelet-Ridgelet denoising technique was introduced. The proposed method was applied on synthetic and natural images. The performance of the combined method was compared with BayesShrink Wavelet image denoising. The experimental results by the proposed method showed the improvement of the visual image quality and increase of SNR in comparison with BayesShrink Wavelet technique especially in the straight regions of the image. Future work is conducted to improve the performance of this method. As the future work the proposed combined method would be considered for image coding and restoration.

6. REFERENCES

[1] C. Taswell, "The what, how, and why of wavelet shrinkage denoising," *IEEE Journal Computing in Science and Engineering,* **2**(3), pp. 12–17, 2000.

[2] D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation via wavelet shrinkage," *Biometrika,* **81**(1), pp. 425–455, Sept 1994.

[3] S. G. Chang, B. Yu, and M. Vetterli, "Adaptive wavelet thresholding for image denoising and compression," *IEEE Trans. on Image Processing,* **9**(9), pp. 1532–1546, 2000.

[4] E. J. Candes, *Ridgelets: Theory and Applications*, Ph.D. thesis, Department of Statistics, Stanford University, 1998.

[5] E. J. Candes and D. L. Donoho, "Ridgelets: a key to higher dimensional intermittency?," *Phil. Trans. R. Soc. Lond. A.* , pp. 2495–2509, 1999.

[6] M. N. Do and M. Vetterli, "The finite ridgelet transform for image representation," *IEEE Tran. on Image Processing,* **12**(1), pp. 16–28, 2003.

[7] N. N. Kachouie, P. Fieguth, and E. Jernigan, "Bayesshrink ridgelet for image denoising," *Lecture Notes on Computer Science,* **3211**(1), pp. 163–170, 2004.

[8] S. G. Chang, B. Yu, and M. Vetterli, "Adaptive wavelet thresholding for image denoising and compression," *IEEE Trans. on Image Processing,* **9**(9), pp. 1532–1546, 2000.

[9] D. L. Donoho and M. R. Duncan, "Digital curvelet transform: Strategy, implementation and experiments," *Proc. SPIE,* **4056**, pp. 12–29, 2000.

[10] J. L. Starck, E. J. Candes, and D. L. Donoho, "The curvelet transform for image denoising," *IEEE Tran on Image Processing,* **11**(6), pp. 670–684, 2002.