MODELING AND PERFORMANCE ANALYSIS OF INITIAL CONNECTION IN IEEE 802.16 PMP NETWORKS[1]

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ABSTRACT

In this paper, we propose an accurate analytical model to analyze the performance of initial ranging requests in IEEE 802.16 networks. Two metrics, connection probability and average connection delay, are investigated to evaluate the network performance. Performance observation demonstrate that the connection probability is not heavily influenced by the contention window size and reconnection retry limitation but the average connection delay is sensitive to the above two parameters. Moreover, we find that improving the service capacity and buffer size of base station can optimize the connection probability and average connection delay.

1. INTRODUCTION

IEEE has been working on the standard of IEEE 802.16 for medium access control layer(MAC) and physical layer specifications for wireless metropolitan area networks (WMANs)[1]. The goal of WMANs is to provide high-speed wireless Internet access. The MAC layer of IEEE 802.16 supports a primarily Point-to-MultiPoint (PMP) architecture. The PMP architecture consists of two kinds of fixed (non-mobile) stations: subscriber station (SS) and base station (BS). The BS regulates all the communication in the network. Each SS can deliver voice and data using common interfaces [2][3].

The communication path between an SS and the BS has two directions: uplink, from the SS to the BS, and downlink, from the BS that may reach many SSs. Both uplink and downlink channels are structured into frames. In the uplink subframe, there is initial maintenance period. In this period, ranging requests are sent by the SS at the initialization phase and periodically thereafter. The BS uses such request to determine network delay and request power or downlink burst profile changes. In this period also new stations may join the network. Since several SSs can access the channel simultaneously, collisions can occur in this period.The mandatory method of contention resolution for initial ranging is based on a truncated binary exponential backoff, with an initial backoff widow size and a maximum backoff window controlled by the BS [4].

The major contribution of this paper is that we propose an accurate analytical model to analyze the performance of initial ranging requests in IEEE 802.16 networks. Two metrics, connection probability and average connection delay, are investigated to evaluate the network performance. To the best of our knowledge, this work is the first one investigating the two metrics of initial ranging in IEEE 802.16 network.

2. FERFORMANCE ANALYSIS

In this section, we study the initial ranging of the networks with a Markov model and give the expressions of connection probability and connection delay as functions of the contention window size.

2.1 Assumptions

Without loss of generality, the following assumptions are used in our analytical model.

1) The BS service capacity is m, i.e., the BS can accept at most m ranging connection requests simultaneously. When a new ranging request arrives, if the connection channel is fully occupied and the total number of ranging request is more than $m+2$, then the new ranging request must enter into the truncated binary exponential backoff state.

2) The buffer size of the BS is N. That is, when a new ranging request arrives, if there are N ranging requests entering the contention resolution process, the new ranging request will be rejected. Without loss of generality, we assume $N>m$.

3) The arrival of ranging requests follows Poisson process with rate λ, and the probability distribution of ranging request connection duration is exponential with mean $1/\mu$. Let $\rho = \frac{\lambda}{m\mu} < 1$.

4) The collision probability of a ranging request is independent

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of the number of retry times (if any) but is dependent on the number of contention ranging requests.

2.2. The Analytical Model

With the above assumptions, the connection activities in initial ranging can be modeled by a three-dimensional Markov Chain $\{s(t), b(t), l(t)\}$, where $s(t)$ is the stochastic process representing the backoff stage $(0,\ldots,r)$ at time t and it records the number of retries a ranging request suffered; $b(t)$ is the stochastic process representing the backoff counter for a given ranging request at time t and records the number of transmission opportunities that the request shall defer before transmitting; $l(t)$ is the number of contending ranging requests at time t.

With the assumptions (1), (2) and (3), the arrival of ranging request can be modeled as *M*/*M*/*m*/*N* systems. According to *M*/*M*/*m*/*N* model [5], we have

$$P_k = \begin{cases} P_0 \cdot \dfrac{(m\rho)^k}{k!} & k \le m \\ P_0 \cdot \dfrac{m^m \rho^k}{m!} & m < k \le N \end{cases} \tag{2.1}$$

Where P_0 is the probability that when a new ranging request arrives, there is idle channel to serve the ranging request and the connection can be established immediately. P_1 denotes the probability that when a new ranging request arrives, the channel is busy and the new ranging request has to wait for the service but there is no other ranging request contenting the channel. P_i denotes the probability that when a new ranging request arrives, the channel is busy and there are i-1 ranging requests contenting the channel. Therefore the new ranging request must enter into the binary exponential backoff process.

Fig. 1 shows the overall Markov chain of the ranging requests. When a new ranging request arrives, it will encounter the competitors and the corresponding probability can be determined by (2.1). Note that the state s in Fig. 1 is a pseudo-state and it is introduced for the convenience of presentation. Fig. 2 shows the detailed Markov chain for k competitors (denoted as $\boldsymbol{M}_k$, including the new initial connection request) where the collision probability C_k depends on the number of competitors k, and is assumed to be a constant.

Based on Fig. 1 and Fig. 2, the only non null one-step transition probabilities in this Markov chain are

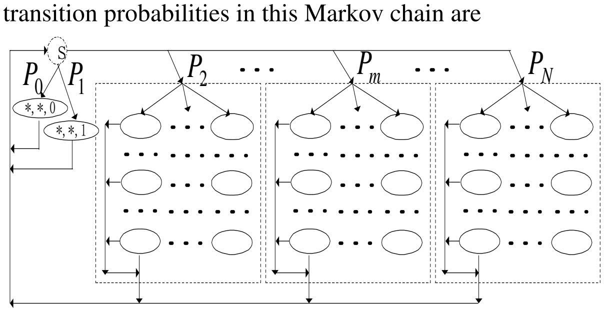

Fig.1 Overall Markov chain model for the ranging connections

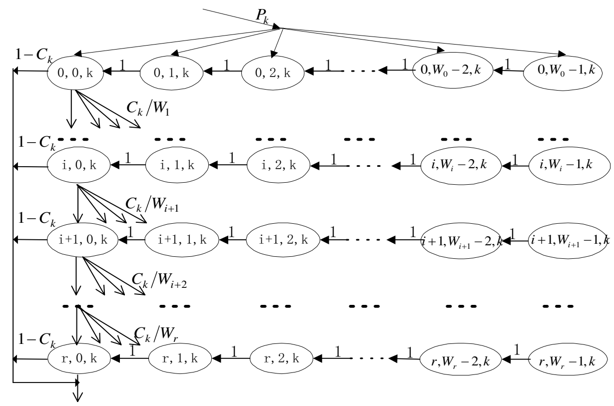

Fig.2. $\boldsymbol{M}_k$ - Markov chain model for the backoff window size for *k* ranging competitors

$$\begin{cases} P\{i, j, k \mid i, j+1, k\} = 1 & i \in [0, r],\ j \in [0, W_i - 2], k \in [2, N] \\ P\{i, j, k \mid i-1, 0, k\} = \dfrac{C_k}{W_i} & i \in [1, r],\ j \in [0, W_i - 1], k \in [2, N] \\ P\{s \mid i, 0, k\} = C_k & i \in [0, r-1], k \in [2, N] \\ P\{s \mid r, 0, k\} = 1 & k \in [2, N] \\ P\{0, j, k \mid s\} = P_k & j \in [0, W_0 - 1], k \in [2, N] \\ P\{*,*, 0 \mid s\} = P_0 & \\ P\{*,*, 1 \mid s\} = P_1 & \end{cases} \tag{2.2}$$

The first equation in (2.2) accounts for the fact that when the SS receives a transmission opportunity, the backoff time counter is decremented. The second equation models the system after an unsuccessful ranging request. When an unsuccessful ranging request occurs at backoff stage i-1, the backoff stage increases and the new backoff value is uniformly chosen in the range [0, W_i-1]. The third and fourth equations account for the fact that once the backoff stage reaches the maximum backoff stage r, after a successful raging request or the rejection of the ranging request, the system is waiting for a new ranging request (return to state s). P_k denotes the probability of k ranging request competitors. The last two equations account for the fact that when the new ranging request arrives, the channel is not fully occupied or even the channel is busy, there are no other ranging competitors to contend the channel.

Let $b_{i,j,k} = \lim_{t\to\infty} P\{s(t)=i, b(t)=j, l(t)=k\}, i \in [0,r]$, $j \in [0, W_{i-1}], k \in [2, N]$ be the stationary distribution of the Markov chain. We can immediately achieve

$$b_{0,0,k} = P_k \tag{2.3}$$

By using local balance equation for each stage in Fig. 1 and Fig. 2, the probability of ranging request in state $\{i+1,0,k\}$ can be deduced from the probability of ranging request in state $\{i,0,k\}$ as below:

$$b_{i+1,0,k} = C_k \cdot b_{i,0,k} \qquad 0 \le i < r \tag{2.4}$$

Thus, (2.4) yields $\quad b_{i,0,k} = C_k^i \cdot b_{0,0,k} \quad 0 < i \le r$

Due to the regularities of the chain, for each $j \in [0, W_i - 1]$, $b_{i,j,k}$ becomes

$$b_{i,j,k} = \frac{W_i - j}{W_i} \cdot \begin{cases} P_k & i = 0 \\ C_k \cdot b_{i-1,0,k} & 0 < i \le r \end{cases} \quad (2.5)$$

By means of relations (2.3) and (2.4), (2.5) can be rewritten as

$$b_{i,j,k} = \frac{W_i - j}{W_i} b_{i,0,k} \text{ for } i \in [0, r] \;\; j \in [0, W_i - 1] \quad (2.6)$$

Thus, by relations (2.1) (2.4) and (2.6), all the values $b_{i,j,k}$ are expressed as functions of the value P_0 and the collision probability C_k. Applying the normalization condition, we have

$$\begin{aligned}
1 &= P_0 + P_1 + \sum_{k=2}^{N}\sum_{i=0}^{r}\sum_{j=0}^{W_i-1} b_{i,j,k} \\
&= P_0 + m\rho P_0 + \sum_{k=2}^{m} P_0 \frac{(m\rho)^k}{k!} \sum_{i=0}^{r} b_{i,0,k} \sum_{j=0}^{W_i-1} \frac{W_i - j}{W_i} + \sum_{k=m+1}^{N} P_0 \frac{m^m \rho^k}{m!} \sum_{i=0}^{r} b_{i,0,k} \sum_{j=0}^{W_i-1} \frac{W_i - j}{W_i} \\
&= P_0 + m\rho P_0 + \sum_{k=2}^{m} P_0 \frac{(m\rho)^k}{k!} \sum_{i=0}^{r} b_{i,0,k} \frac{W_i + 1}{2} + \sum_{k=m+1}^{N} P_0 \frac{m^m \rho^k}{m!} \sum_{i=0}^{r} b_{i,0,k} \frac{W_i + 1}{2} \\
&= P_0 + m\rho P_0 + \sum_{k=2}^{m} P_0 \frac{(m\rho)^k}{k!} \sum_{i=0}^{r} C_k^i \frac{W_i + 1}{2} + \sum_{k=m+1}^{N} P_0 \frac{m^m \rho^k}{m!} \sum_{i=0}^{r} C_k^i \cdot \frac{W_i + 1}{2} \\
&= P_0 \left[1 + m\rho + \sum_{k=2}^{m} \frac{(m\rho)^k}{k!} \sum_{i=0}^{r} C_k^i \frac{W_i + 1}{2} + \sum_{k=m+1}^{N} P_0 \frac{m^m \rho^k}{m!} \sum_{i=0}^{r} C_k^i \frac{W_i + 1}{2} \right]
\end{aligned}$$

Let $G(k) = \sum_{i=0}^{r} C_k^i \frac{W_i + 1}{2}$,the above formula can be presented as

$$1 = P_0 \left[1 + m\rho + \sum_{k=2}^{m} \frac{(m\rho)^k}{k!} G(k) + \sum_{k=m+1}^{N} \frac{m^m \rho^k}{m!} G(k) \right] \quad (2.7)$$

Once we derive P_0 and C_k, the stationary probability of any state (i,j,k) can be calculated by the above equations. We can now express τ_k, which denotes the probability that the ranging request in a randomly chosen slot time contending with k-1 other ranging competitors. As any initial ranging transmission occurs when the backoff time counter is equal to zero, regardless of the backoff stage, it is

$$\tau_k = \sum_{i=0}^{r} C_k \cdot b_{i,0,k} = \sum_{i=0}^{r} C_k^i b_{0,0,k} = \sum_{i=0}^{r} C_k^i \cdot \begin{cases} \frac{(m\rho)^k}{k!} P_0 & k \le m \\ \frac{m^m \rho^k}{m!} P_0 & m < k \le N \end{cases}$$

$$= \begin{cases} \frac{1 - C_k^{m+1}}{1 - C_k} \cdot \frac{(m\rho)^k}{k!} P_0 & k \le m \\ \frac{1 - C_k^{m+1}}{1 - C_k} \cdot \frac{m^m \rho^k}{m!} P_0 & m < k \le N \end{cases} \qquad 2 \le k \le N \quad (2.8)$$

In (2.8), τ_k depends on the conditional collision probability C_k. It is sufficient to note that C_k is the probability that, in a time slot, at least one of the k-1 remaining connection requests is trying to access the channel. Each remaining ranging request accesses to the channel with probability C_k. This yields

$$C_k = 1 - (1 - \tau_k)^{k-1} \qquad 2 \le k \le N \quad (2.9)$$

It can be seen that (2.7) (2.8) and (2.9) form a nonlinear equation systems with unknown variable P_0, τ_k and C_k (i=2,…,N), which can be solved by numerical method.

2.3 Analyses of Connection Probability and Delay

Let CP be the connection probability of the network. It is defined as the probability that a ranging request is accepted in a given network. The unsuccessful connection probability is due to two situations: (1) system busy probability (UP_1) when there are N ranging requests waiting in the queue of BS and the channel is busy at the moment, the ranging request gets lost; and (2) Max backoff stage probability (UP_2) when the ranging request reaches the maximum stage r, it will be rejected if it still collides with other ranging requests. Since P_0 can be solved in the nonlinear equation systems (2.7) (2.8) and (2.9) and UP_1 is equal to the probability that there are N ranging requests in the system, according to (2.1), we obtain

$$UP_1 = P_N = P_0 \cdot \frac{m^m \rho^N}{m!}$$

Still according to (2.1), solving for P_0 and C_k (k=2,…,N) in the nonlinear equation systems (2.7) (2.8) and (2.9), we obtain

$$UP_2 = \sum_{k=1}^{N} P_k \cdot C_k^{r+1} = \sum_{k=2}^{m} (P_0 \cdot \frac{(m\rho)^k}{k!} \cdot C_k^{r+1}) + \sum_{k=m+1}^{N} (P_0 \cdot \frac{m^m \rho^k}{m!} \cdot C_k^{r+1})$$

UP_1 and UP_2 are the reject probabilities of ranging request. Therefore the connection probability is

$$\begin{aligned} CP &= 1 - (UP_1 + UP_2) \\ &= 1 - [P_0 \cdot \frac{m^m \rho^N}{m!} + \sum_{k=2}^{m} (P_0 \cdot \frac{(m\rho)^k}{k!} \cdot C_k^{r+1}) + \sum_{k=m+1}^{N} (P_0 \cdot \frac{m^m \rho^k}{m!} \cdot C_k^{r+1})] \quad (2.10) \end{aligned}$$

Let CD be the average connection delay for ranging requests, defined as average number of slots times that a ranging request suffers from entering the network to the establishment of the connection. We can model the number of connection times as geometrically distributed with success probability 1- C_k. The expected backoff window size is

$$\begin{aligned} W_k^{backoff} &= (1 - C_k)\frac{W}{2} + C_k(1 - C_k)\frac{2W}{2} + ... + C_k^r (1 - C_k)\frac{2^r W}{2} \\ &= \frac{(1 - C_k)(1 - (2C_k)^{r+1})}{1 - 2C_k} \cdot \frac{W}{2} \end{aligned}$$

Therefore, CD can be calculated according to the expected backoff window size for M_k

$$CD = \sum_{k=2}^{N} P_k \cdot W_k^{backoff} = \sum_{k=2}^{m} \left[P_0 \cdot \frac{(m\rho)^k}{k!} \cdot \frac{(1 - C_k)(1 - (2C_k)^{r+1})}{1 - 2C_k} \cdot \frac{W}{2} \right] + \sum_{k=m+1}^{N} \left[P_0 \cdot \frac{m^m \rho^k}{m!} \cdot \frac{(1 - C_k)(1 - (2C_k)^{r+1})}{1 - 2C_k} \cdot \frac{W}{2} \right] \quad (2.11)$$

3. EXPERIMENTAL EVALUATION

In this Section, we report simulation results and make observations. We assume that the ranging requests for connection establishment form a Poisson process with λ, while the connection lifetimes are exponentially distributed with average of 10 seconds.

Fig. 3 shows the effect of the service capacity of the network to the connection probability and connection delay under the parameters $\{W_{min}=32, W_{max}=1024, N=100\}$. The arrival rate of ranging request ranges from 2.5 to 30.0. It is observed that the connection probability decreases while the connection delay increases with the increase of ranging request arrival rate. However, under the same parameters, when the service capacity of the network increases, the connection probability improves sharply and the connection delay drops quickly.

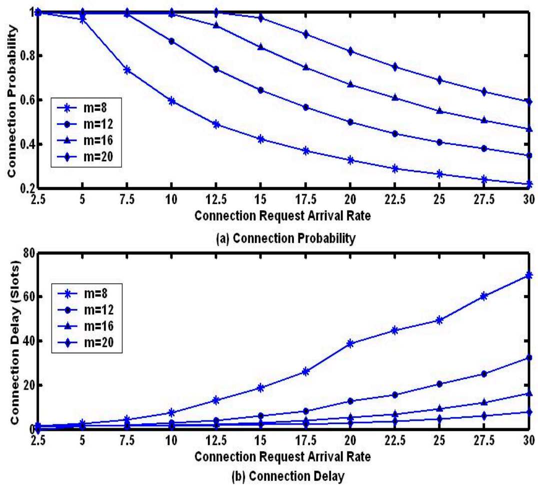

Fig. 3 Effect of the service capacity

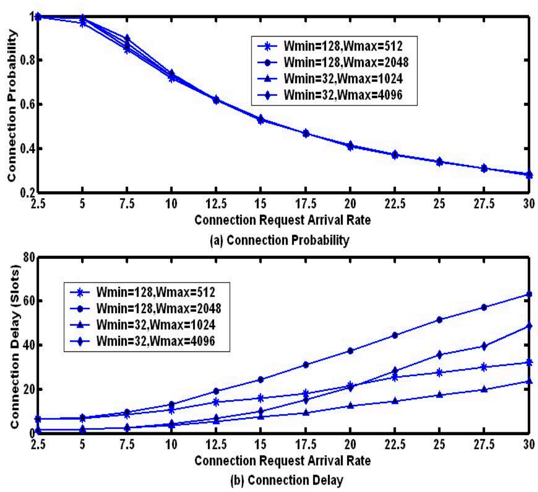

Fig. 4 Effect of contention windows

Fig. 4 shows the effect of contention window size to the two metrics when $m=10$, $N=100$. From Fig 4(a), it can be seen that the connection probability is very close for all cases, especially when the ranging request arrival rate is high. When the ranging request arrival rate is low, there is a little difference in the four cases. With the same minimal contention window size, the larger maximum backoff stage results in a higher connection probability. With the same maximum backoff stage, the lower contention window also brings a higher connection probability. Fig 4(b) demonstrates that for the same minimal contention window size, the less the maximum backoff stage is, the lower is the average connection delay. For the same maximum backoff state setting, the lower contention window size results in lower connection delay.

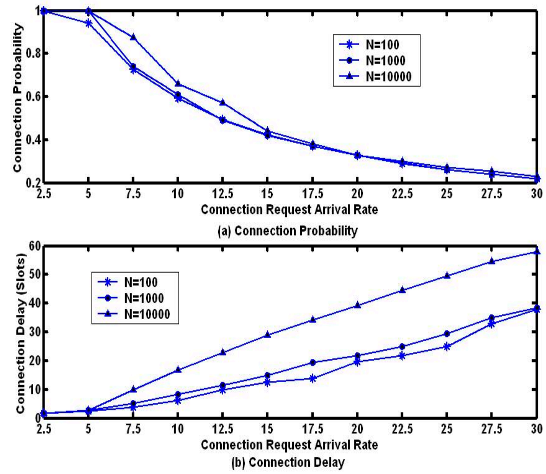

Fig. 5 Effect of buffer size to

Fig. 5 shows the effect of buffer size to the two metrics under the parameters $\{W_{min}=32, W_{max}=1024, m=15\}$. These figures also show that increasing the buffer size can improve the connection probability whereas increases average connection delay.

4.CONCLUSIONS

We have proposed an analytical model to analyze the initial ranging connection activity for IEEE 802.16 networks. Two performance metrics (connection probability and connection delay) are provided to evaluate the IEEE 802.16 PMP networks. The performance observation demonstrates that the connection probability is not heavily influenced by the contention window size and reconnection retry limitation but the connection delay is sensitive to the contention window size and retry limitation. Improving the service capacity and the buffer size of the BS can optimize the connection probability and reduce the average connection delay.

5. REFERENCES

[1] IEEE 802.16-2004 “Part 16: Air Interface for Fixed Broadband Wireless Access Systems”, Oct. 2004.

[2] C. Eklund, R. B. Marks, K. L. Stanwood and S. Wang, “IEEE Standard 802.16a: A Technical Overview of the WirelessMAN™ Air Interface for Broadband Wireless Access.” *IEEE Communications Magazine*, vol. 40, no. 6, pp. 98–107, Jun. 2002.

[3] S. J. Vaughan-Nichols, “Achieving Wireless Broadband with WiMax”, *IEEE Computer*, vol. 37, no. 6, pp. 10–13, June 2004.

[4] Dong-Hoon Cho, Jung-Hoon Song, Min-Su Kim, and Ki-Jun Han, “Performance Analysis of the IEEE 802.16 Wireless Metropolitan Area Network”, Proc. of the First International Conference on Distributed Frameworks for Multimedia Applications, 2005.

[5] Dimitri Bersekas and Robert Gallager, Data Netoworks, New Jerse, pp.134–138, 1987.