

Modeling Magnetic Coupling for On-Chip Interconnect[†]

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Abstract—As advances in IC technologies and operating frequencies make the modeling of on-chip magnetic interactions a necessity, it is apparent that extension of traditional inductance extraction approaches to full-chip scale problems is impractical. There are primarily two obstacles to performing inductance extraction with the same efficacy as full-chip capacitance extraction: 1) neglecting far-away coupling terms can generate an unstable inductance matrix approximation; and 2) the penetrating nature of inductance makes localized extraction via windowing extremely difficult. In this paper we propose and contrast three new options for stable and accurate window-based extraction of large-scale magnetic coupling. We analyze the required window sizes to consider the possibilities for pattern-matching style solutions, and propose three schemes for determining coupling values and window sizing for extraction via on-the-fly field solution.

Index Terms—Inductance, Susceptance, Magnetic Interaction, Interconnect Modeling.

I. Introduction

THE magnetic interactions of on-chip interconnects are becoming more important due to the denser circuits and higher operating frequencies. This is captured in terms of inductance, which is a property of closed-loop currents. Calculating loop inductance is a chicken-egg problem since finding the currents to define the loops requires the inductance to find the currents. Partial inductance [12] is the general solution for such problems since it evaluates the inductance for each wire *segment* as it forms an imaginary loop with infinity.

But building a complete partial inductance matrix for all of the interconnects on an IC is impractical, and arbitrarily discarding coupling terms to sparsify the partial inductance matrix can lead to instability [1]. Based on this increased complexity and what we know from full-chip capacitance extraction, any attempt at full-chip inductance extraction requires some form of localizing via windowing. However, to provide for windowing in a stable and accurate manner is far more challenging for inductance than it is for capacitance due to the penetrating nature of the magnetic fields.

Modeling the partial susceptance¹ — the inverse of partial inductance — directly was recently proposed [6] due to its analogous behavior to capacitance. For example, the inverse of a partial inductance matrix is diagonally dominant and can be sparsified easily in a stable manner [3]. There-

fore, upon generation of a sparse susceptance model, one could use the susceptances directly in a simulator which is modified to support such elements [6], or invert the sparse susceptance matrix to generate a sparse, stable inductance approximation [3]. The complexity of both approaches is dominated by the inversion of the full partial inductance matrix of magnetic vector potentials to generate the partial susceptance matrix. Shell methods [13][14] provide some advantage over both in that the sparsification is performed as part of the analysis of the magnetic vector potentials, and does not require the full partial inductance matrix. But using windowing we can exploit properties of susceptance for localized regions, as long as the models for the regions can be combined to form one complete, stable approximation for the entire system.

Even if pattern-matching for generalized inductance extraction is impractical due to the number of unique patterns and the nature of magnetic fields [4], full-chip solutions require windowing to create more manageable field solution problems. In this paper we will show that piecing together solutions in the susceptance domain can be done in a manner that provably generates a stable complete-system susceptance or inductance approximation. We will further show a new approximation for direct sparse inductance approximation based on a modified magnetic vector potential definition — as a function of window size — that results in an equally effective alternative to inverting the initial inductance matrix. Results are shown to compare and contrast these approximations.

While all of these methods and models appear to be promising, the greatest challenge for all of these approaches, including the aforementioned shell methods, is determination of the proper window size. With inductance, if the design is such that currents return only at large distances, then *localized* magnetic models (no matter if inductance or susceptance is used) cannot resolve the loop coupling. In addition, since magnetic couplings are very small between orthogonal conductors, there is minimal shielding and significant interaction between layers that are weakly coupled capacitively, such as M3 and M5 or M4 and M6, etc. We analyze this problem for on-chip structures and demonstrate the need for larger inductance extraction windows to provide accuracy comparable to what is obtained for full-chip window-based capacitance extraction.

II. Windowing Based on Susceptance Properties

A. Windowing for Inductance or Susceptance Extraction

A potential extraction flow that we are proposing consists of the following steps:

1. Historically, the name of the inverse of inductance is the *reluctance*, but ‘susceptance’ seems more appropriate, since ‘an influence can be *induced* easily if the counterpart is *susceptive* to it’.

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1. Generate a partial inductance matrix $L^{(j)}$ for active conductor j and all conductors within a window around it.
2. Solve $L^{(j)}$ to find the currents flowing through these conductors such that the total flux for the active conductor j through its loop with infinity is unity, and zero for the rest. The currents are equal to the susceptive couplings $S^{(j)}_{ij}$ between the active conductor j and the group of conductors within the window around it².
3. Repeat steps 1 and 2 for all conductors in turn as active conductors.
4. Merge all $S^{(j)}$ vectors into one complete, sparse susceptance matrix that approximates the magnetic interactions for the entire interconnect system.
5. The combined susceptance matrix will be asymmetric, since $S^{(j)}_{ij}$ is not necessarily equal to $S^{(i)}_{ji}$. By choosing the value with the *smaller magnitude* for both entries, we can prove that the complete susceptance model can be rendered symmetric and stable (see Subsection II.C.).
- 6a. *Option 1:* Invert the sparse, symmetric susceptance matrix to generate an inductance matrix. The truncation in the susceptance domain makes this matrix much easier to sparsify than the original inductance model. This *double-inverse inductance matrix* can then be used for timing analysis or simulation without loss of generality (see Section III).
- 6b. *Option 2:* Use susceptance directly for simulation. Requires a simulator that supports susceptance [6] (see Section IV).

Before we show examples and compare these approaches, we begin with some background on the properties that facilitate this extraction methodology.

B. Properties of Magnetic Interaction Matrices

We assume a boundary element approach where the current density is constant for each filament. A_i is the average vector potential for filament i in field direction α , which is x , y or z . The *length-current* of filament j is $\mu I_j l_j$ which we denote with γ_j where I_j is the current for this section and l_j its length. We can write the discretized magnetic interconnect interactions as

$$A \approx L\gamma \text{ with } L_{ij} \equiv \frac{1}{V_i} \frac{1}{V_j} \frac{1}{4\pi} \int_{V_i} \int_{V_j} \frac{d\vec{r}_i \cdot d\vec{r}_j}{\|\vec{r}_i - \vec{r}_j\|} da_i da_j \quad (1)$$

by defining the matrices L^α . V_i is the filament volume of i . Please note that L is *not* the commonly used inductance matrix but rather inductance normalized by the lengths of the filaments involved. We will refer to it as inductance in the following. The inverse of this inductance matrix is the *susceptance matrix* S of the system.

It can be easily shown (see [3]) that both L and S (according to definition above) are positive definite, that S has positive diagonal and non-positive off-diagonal elements, and that S (similar to capacitance C) is diagonally dominant (magnitude of diagonal larger as sum of magnitudes of off-diagonals). S remains diagonal dominant even if off-diagonal

nals are set to zero. With the following theorem from linear algebra [11][pg. 349] for a matrix A :

$$\text{If all } A_{ii} > 0 \text{ and } A, A^T \text{ diagonal dominant} \\ \text{then } A \text{ is positive definite.} \quad (2)$$

we find that S remains positive definite even if truncated — as opposed to L which usually does not (see [1]).

To assist in understanding the physical interpretation of susceptance, we focus on the description of the j^{th} column of S . This column represents the amount of current necessary *on or flowing through* the conductors to force conductor j to unit magnetic potential and all other conductors to zero. An individual term S_{ij} , when i and j are far removed, must include shielding effects for the magnetostatic fields. The ‘current necessary’ in some conductor i to force it to zero potential already takes into account that some of the original field of the reference (unit potential) conductor j has been compensated by currents on zero potential conductors closer to j . This in turn means that the magnitude of the elements in S_{ij} drops off much faster with distance between i and j . This enhances sparsification, since elements which are ‘large enough’ in S are easier to distinguish from those that are ‘too small’, thereby forming a much smaller set than for the inductance matrix L .

C. Windowing and Forcing Stable Symmetry of S

Since the shielding effect renders all but a few short-distance couplings negligible, we can exploit this to make the inversion from L to S more efficient using windowing. By restricting the area of interest to *extraction windows* around each conductor, we replace the inversion of a huge, dense $N \times N$ matrix — N being the total number of conductors in the system — by N much smaller, $n_j \times n_j$, matrix inversion problems. This results in N individual, small matrices $S(j)$, where n_j is the number of conductors to which segment j has significant couplings.

The $S(j)$ are all diagonally dominant, since the proof above applies to each of the small conductor subsets individually. Clearly, for this approach the coupling $S_{ij}(j)$ need not be equal to $S_{ji}(i)$, since the set of ‘significant neighbors’ within a window is usually different for different segments i and j . However, when we assemble our sparse S' matrix for the entire system from the individual $S(j)$, we need to ensure symmetry of S' to guarantee positive definiteness. Therefore, we choose

$$S'_{ij} = S'_{ji} = \min\{S_{ij}(j), S_{ji}(i)\} \quad (3)$$

Since we know that all off-diagonal elements of any inverse interaction matrix are non-positive, this means we select the element with the *smallest magnitude*. This ensures the diagonal dominance of S' when assembling it from elements of the $S(j)$, while preserving the largest degree of accuracy. With (2) we then find that the sparse approximation S' is positive definite. This selection of the smallest magnitude off-diagonal is simple yet critical aspect of ensuring the stability of the resulting susceptive model.

III. Double-Inverse Inductance Models

Using S directly in simulators and timing analysis tools [6] is not readily supported today. The global, sparse susceptance matrix is positive definite, therefore, inverting this S matrix back into an inductance representation produces a

2. We use S for susceptance, since K (as used in [6]) can cause confusion with mutual inductance for which K is used in SPICE.

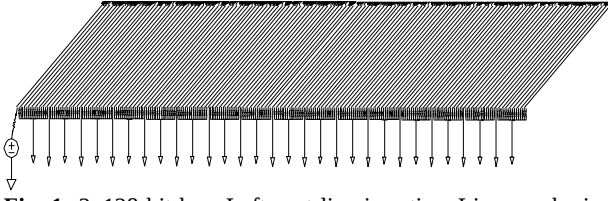


Fig. 1: 2x128 bit bus. Leftmost line is active. Line numbering from left to right.

positive definite L matrix. This matrix would be less sparse than S since the inverse of a sparse matrix is not necessarily sparse.

In addition, for susceptance the window size was chosen to include only long-range susceptible couplings above a given magnitude threshold relative to the self terms. One percent cutoff means, for instance, that the window size was chosen such that increasing the window only added new susceptible couplings less than 1% of the self term for a given active conductor. The window sizes necessary for an equally accurate inductance window would tend to be larger, due to the lack of shielding that is at work for S .

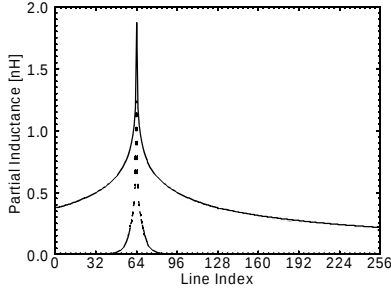


Fig. 2: Comparison of original inductive couplings with double-inverse L . Shown are couplings of all bus lines of example in Fig. 1 to line 64 (middle of first block). Solid line for original partial inductance, dotted line for double-inverse L (all elements of susceptance matrix smaller than 1 % of maximum were discarded).

But the resulting double-inverted partial inductance matrix generally contains far fewer significant elements than the original inductance matrix, making further sparsification of L much easier (see Fig. 2). To preserve positive definiteness of the doubly inverted inductance matrix, we add the magnitude of off-diagonals which we cancel during sparsification to the corresponding diagonal elements. That this procedure preserves positive definiteness is easy to show. We define a symmetric matrix $M_{ij}(p,q)$ which is +1 for $i=j=p$, and $i=j=q$, either +1 or -1 for both $(i=p,j=q)$ and $(i=q,j=p)$, and 0 everywhere else.

$x^T M(p,q)x$ is always greater or equal to zero, which can be shown by explicitly calculating the expression. If a symmetric matrix A is positive definite, then the matrix $B = A + |A_{pq}| * M(p,q)$ must be positive definite as well. If we choose the sign of the off-diagonals in $M(p,q)$ opposite to the sign of A_{pq} , then $B_{pq} = B_{qp} = 0$.

Since most of the off-diagonal terms of the double-inverse inductance matrix are very small (see Fig. 2), this procedure ensures the positive definiteness of the resulting sparse double-inverse inductance matrix while not significantly changing the diagonal elements. We use the same cutoff percentage threshold for the double-inverse inductance matrix as for the susceptance matrix. This sparse inductance model can efficiently and accurately represent

magnetic interactions within interconnect without compromising stability. The need to perform two inversions to generate the double-inverse matrix, however, increases the extraction time. This issue we attempt to remove with the direct sparse inductance technique presented in Section VI.

IV. Direct Simulation of Susceptance

Susceptance S and resistance R of a segment are connected in series as shown in Fig. 3. In the following we will derive the Norton companion models for susceptance elements for backward Euler and trapezoidal time integration within a circuit simulation tool. Fig. 4 shows the schematic for the Norton equivalents for two coupled self susceptance elements:

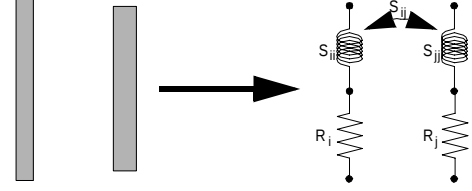


Fig. 3: Single stage RS model for two magnetically coupled segments.

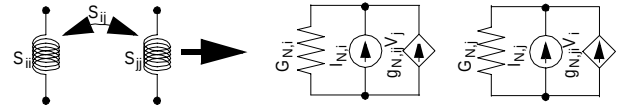


Fig. 4: Norton Companion Model for Susceptance Elements.

The core difference from a circuit perspective is that mutual susceptances contribute *voltage controlled sources* to the companion models, rather than current controlled sources as for the mutual inductors. The concept for this has been presented in [6] without, however, explicitly developing the coefficients for the companion models. We will show this derivation in the following. Starting point is the current-voltage relationships for the susceptances involved. The equation

$$\partial_t I_i = S_{ii} V_i + S_{ij} V_j \quad (4)$$

is easily established by inverting the corresponding and well-known relation for inductive couplings:

$$V_i = L_{ii} \partial_t I_i + L_{ij} \partial_t I_j \quad (5)$$

Note that (4) and (5) represent 2×2 matrix equations and can be readily generalized for $N \times N$ systems. For numerical time integration we need to discretize Eq. (4) over the continuous time t by integrating the equation over one time step:

$$I_i(t + \Delta t) - I_i(t) = S_{ii} \int_t^{t+\Delta t} V_i(\tau) d\tau + S_{ij} \int_t^{t+\Delta t} V_j(\tau) d\tau \quad (6)$$

On the left hand side of (6) the integration of the time derivative of the induced current in segment i leads to a difference expression of the current at the two adjacent time-points. This leads to the Norton current equation:

$$I_i(t + \Delta t) = I_i(t) + S_{ii} \int_t^{t+\Delta t} V_i(\tau) d\tau + S_{ij} \int_t^{t+\Delta t} V_j(\tau) d\tau \quad (7)$$

If we approximate the voltage integrals in Eq. (7) by the voltage values at the later timepoint

$$\int_t^{t+\Delta t} V_i(\tau) \approx \Delta t V_i(t + \Delta t) \quad (8)$$

we get the Norton equation for backward Euler time integration:

$$I_i(t + \Delta t) = I_i(t) + S_{ii}\Delta t V_i(t + \Delta t) + S_{ij}\Delta t V_j(t + \Delta t) \quad (9)$$

In Eq. (9) each term on the right hand side maps to elements in the Norton equivalent model shown in Fig. 4. The resulting coefficients for the conductance and the current sources are for backward Euler

$$G_{N,i} = S_{ii}\Delta t; I_{N,i} = I_i(t); g_{N,ij} = S_{ij}\Delta t \quad (10)$$

On the other hand, if we approximate the voltage integrals in Eq. (7) by the average of the voltage values at both timepoints

$$\int_t^{t+\Delta t} V_i(\tau) \approx \frac{\Delta t}{2} [V_i(t + \Delta t) + V_i(t)] \quad (11)$$

we get the Norton equation for trapezoidal time integration:

$$I_i(t + \Delta t) = \left[I_i(t) + \frac{S_{ii}\Delta t}{2} V_i(t) + \frac{S_{ij}\Delta t}{2} V_j(t) \right] + \frac{S_{ii}\Delta t}{2} V_i(t + \Delta t) + \frac{S_{ij}\Delta t}{2} V_j(t + \Delta t) \quad (12)$$

Again, each term on the right hand side maps to elements in the Norton equivalent model shown in Fig. 4. The resulting coefficients for the conductance and the current sources are for trapezoidal rule equal to

$$G_{N,i} = \frac{\Delta t}{2} S_{ii}; I_{N,i} = I_i(t) + \frac{\Delta t}{2} [S_{ii} V_i(t) + S_{ij} V_j(t)]; g_{N,ij} = \frac{\Delta t}{2} S_{ij} \quad (13)$$

Implementing this in a prototype Matlab simulator showed stability of the truncated susceptance model where the truncated inductance model was unstable. The example was a 16 bit bus structure with each line subdivided into four segments:

	Full System	Truncated	Sparsity
Inductance	40.3 s	Unstable	95.5%
Susceptance	48.9 s	4.5 s	98.4%

Table 1: Inductance – Susceptance Performance

V. Capacitance vs. Susceptance Windowing

In Section II we showed that many of the properties which apply to capacitance matrices apply to susceptance as well. Among these properties are the shielding effect and the possibility to extract both C and S by solving for the couplings of each active segment to its immediate neighborhood and then assembling all individual results into one sparse and stable global model (windowing).

For capacitance extraction this property is exploited in *pattern matching extractors* which subdivide the layout into very small units containing only very simple metal patterns. Due to the regularity of the on-chip interconnect there is a high degree of *repetition* among these patterns, if the pattern size is only a small multiple of the feature size. Of the millions of patterns which compose the entire layout, only a few thousand may be unique. Only these are computed with field solvers and stored for future retrieval. If, as for capacitance, the important couplings are all within these small patterns, we can model the interactions for the entire layout

very precisely by only extracting the couplings for the relatively few unique patterns.

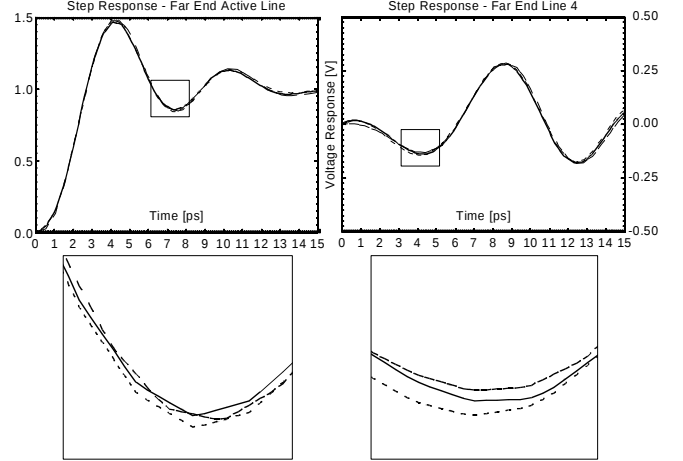


Fig. 5: Accuracy comparison for capacitance with one neighbor left and right (dotted) and susceptance with *twelve* neighbors left and right (dashed). Solid curve is result of full model. Magnification shown below each curve.

Unfortunately, the number of unique interconnect patterns increases very rapidly even for moderate increases of the pattern size, since the small scale repetition disappears very quickly as we move to larger scales [4]. This has been a major problem for modeling inductance, due to its significant long distance interactions. The use of susceptance alleviates this problem somewhat, but as the following will show not enough to make pattern matching as for capacitance feasible.

To demonstrate this we have extracted the full system shown in Fig. 1 using capacitance and susceptance. In Fig. 5 we show the far end voltage responses for the active and the third-next neighboring quiet line for the CS model (solid), full C and truncated S with 25 nonzeros in each row (dashed) and full S plus truncated C with three nonzeros per row (dotted). The accuracy is about equal, even though the sparsity of the capacitance approximation is much higher.

What is the reason for this discrepancy? When capacitance is generated by inverting the potential matrix, we assume when calculating the i^{th} column of the C matrix, that segment i is at unit potential, and *all other conductors are grounded*. This is compatible with the conditions during simulation when quiet lines are actually connected to a constant potential (and can be treated as if they're connected to ground).

For susceptance, each column is generated assuming that the *total flux* for each “quiet” line is zero, and for the “active” line it is unity. During simulation, however, the flux even for actually quiet lines (lines connected to zero potential) can be different from zero, if they happen to be inside of the current return loop of the active line (see Fig. 6).

Therefore, the magnitude of the off-diagonal elements in the susceptance matrix are not a direct reflection of their relative importance. In our example in Fig. 5 the window size is *twelve* times as large for susceptance as for capacitance. But even if the factor were 3x or 4x this would dramatically increase the number of unique patterns for pattern-based extraction, rendering the database approach impractical.

Furthermore, susceptive couplings are, as inductive couplings, very weak between orthogonal conductors. There-

fore the susceptible couplings extend beyond adjacent layers of a chip. Consequently, the average significant coupling distance is much larger for susceptance than for capacitance, which is effectively shielded by the neighboring layers. This contributes further to the increased coupling distance of susceptance with respect to capacitance.

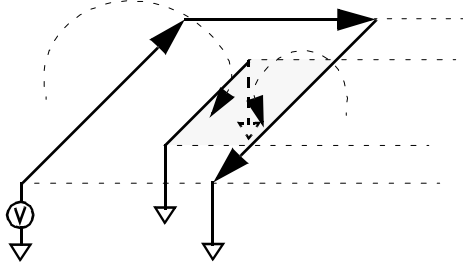


Fig. 6: Line inside current return loop of active line (left). Total flux through middle line is not zero although it is not active.

VI. Direct Sparse Inductance Approximations

From Fig. 2 it is apparent that truncating elements from the susceptance matrix has the roughly following effect on the double-inverse inductance matrix: the original logarithmic distance dependence of the inductive couplings becomes an exponential one. Furthermore, the more off-diagonals we discard from the susceptance matrix (preserving stability), the more pronounced this exponential behavior becomes. It appears as if the point-to-point magnetic potential is no longer proportional to $1/r$ but rather to

$$\|\vec{A}(\vec{r})\| \sim \frac{\exp(-\alpha r)}{r} \quad (14)$$

With α being a *damping factor* of the potential and $\hat{r}$ the unit vector in radial direction in three dimensions.

The more elements that are truncated from the susceptance matrix, the larger α becomes. α has the units of length^{-1} . If we use the potential in Eq. (14) for inductance calculation rather than the usual $1/r$, then α has the meaning of a “window” size of an exponentially smooth “window”. Since the resulting inductance matrices look very much like those gained from the double-inverse procedure described earlier, we expect very accurate results for simulation as well, but *without the requirement of two matrix inversions!* The only remaining issue is how to efficiently calculate inductive couplings for these *exponential potentials*. We can show that Eq. (14) is the solution to

$$(\vec{\nabla}^2 - \alpha^2)A(\vec{r}) = \delta(\vec{r}) \quad (15)$$

by inserting (14) in (15). For an infinite filament in three dimensions (a “point” in two dimensions) Eq. (15) is radially symmetric and becomes

$$\left(\partial_\rho^2 + \frac{1}{\rho}\partial_\rho - \alpha^2\right)A_z(\rho) = \delta(\rho) \quad (16)$$

where ρ is the radius in cylindrical coordinates and A_z the axial component of the magnetic vector potential. This is a scaled modified Bessel equation and its solution of interest (with a singularity at $r = 0$) is

$$\vec{A}(\rho) = K_0(\alpha\rho)\hat{z} \quad (17)$$

where K_0 is a modified Bessel function of second kind and $\hat{z}$ the unit vector in cylindrical axial direction. Very efficient methods are available to evaluate this function (see [10]) which approaches zero logarithmically and infinity exponentially. This solution can be used to compute very good approximations for couplings between parallel long filaments for exponential couplings as well.

Equation (15) reduces to Poisson’s equation for $\alpha = 0$ and it is relatively straightforward to show that the inductance matrix is positive definite by proving that the potential-to-current relationship defined by (15) leads to positive field energies exclusively.

The magnetic field energy is defined as $\mu \int (\vec{A} \cdot \vec{j}) dV$, integrating over the entire space. After discretization this is equivalent to $I^T L I$. For an arbitrary current density, (15) becomes

$$(\vec{\nabla}^2 - \alpha^2)\vec{A}(\vec{r}) = -\mu\vec{j}(\vec{r}) \quad (18)$$

Multiplying (18) with the vector potential and integrating over all space gives

$$\int (\vec{A} \cdot \vec{\nabla}^2 \vec{A}) dV - \alpha^2 \int (\vec{A} \cdot \vec{A}) dV = -\mu \int (\vec{A} \cdot \vec{j}) dV \quad (19)$$

The first integral can be transformed using partial integration, resulting in

$$-\int (\vec{\nabla} \cdot \vec{A})^2 dV - \alpha^2 \int (\vec{A} \cdot \vec{A}) dV = -\mu \int (\vec{A} \cdot \vec{j}) dV \quad (20)$$

since $\vec{A}$ goes to zero as we approach infinity. Since both integrals on the left hand side are zero only if $\vec{A}$ is zero and positive otherwise, we find that for the modified Poisson’s equation (18) the field energy is positive definite, and therefore, our exponential potential inductance matrix is stable.

Using expressions (14) and (17) we can generate the inductive coupling using exponentially damped potentials very efficiently. The resulting L matrix can be sparsified easily, since most of its elements are negligible, due to the exponential decay of the off-diagonal elements with distance. To preserve the stability while truncating the exponential inductance matrix, we add the magnitude of all off-diagonals which we cancel to the corresponding diagonals. Since the off-diagonals decay rapidly with distance, this introduces only a negligible amount of error while keeping the inductive model very compact.

VII. Example

We have applied these models and algorithms to the example in Fig. 1 to compare the results from the double-inverse inductance model with those from the exponential potential approach. In Fig. 7 we see the active line step response for different approaches. Both the exponential potential as well as the double-inverse method presented in this paper are very accurate. Simple truncation diverges soon, due to indefiniteness of the simplified inductance matrix. If only C is truncated, but L is exact, then the results are indistinguishable from the full matrix solution. Fig. 8 shows the voltage responses for line number four in Fig. 1, which is the third-next quiet line from the active wire. The accuracy is a bit lower than for the active line, however, it is sufficient to determine the noise voltage peak values of $[+0.3V; -0.2V]$. The simulation runtimes in Table 2 reflect the efficiency of the proposed inductance modeling

approaches. For the simple truncation run (square) both the number of internally used nodes as well as the relatively long runtime indicate the instability of that particular model. Both the double-inverse approach and the exponential potential method proposed in this paper have much better runtime and memory performance as the full or the truncated system³, while being close to the exact waveforms.

Symbols see Fig. 7:	○	○	●	◆	■
Runtime (step) [s]	19445	16574	101	92	1160
Runtime (ramp) [s]	19423	16561	101	90	1112
Memory [MByte]	114	43.8	8.42	8.43	28.7
Capacitance El.	32782	3275	3275	3275	3275
Inductance El.	32896	32896	6699	6790	6790
Sparsity [%]	0	45	85	85	85
HSpice internal El.	66222	36715	10518	10609	24862

Table 2: Performance for 2x128 bit bus example in Fig. 1.

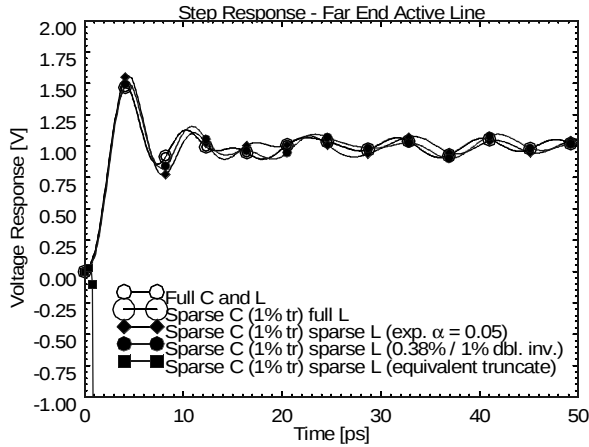


Fig. 7: Voltage response at far end of active line for step input for Fig. 1. Note the divergence of the truncated inductance model. Both double-inverse and exponential potential have similar accuracy.

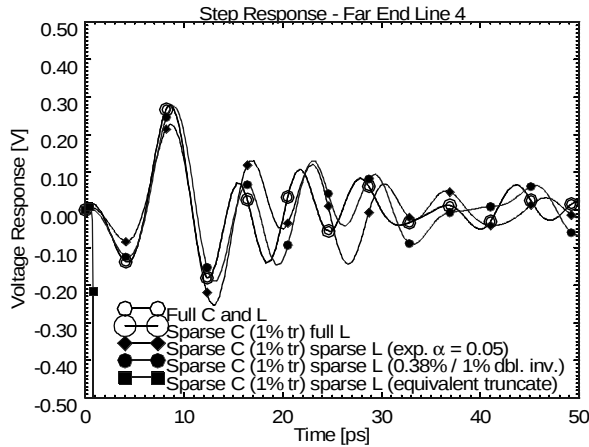


Fig. 8: Voltage response at far end of third-next quiet line for step input for Fig. 1. Note the divergence of the truncated inductance model. Both double-inverse and exponential potential have similar accuracy.

3. The excessively large runtimes for the full system seem to be a problem specific to adaptive timestepping in HSpice, judging from preliminary results with a simulator based on Berkeley Spice 3f5.

VIII. Conclusions

We have presented three approaches to generating stable and sparse yet very accurate inductance models for on-chip magnetic interactions between interconnect wires. We have investigated the similarities and differences of susceptance and capacitance and the possibilities of using susceptance directly for simulation. While susceptance does alleviate some of the shortcomings of the traditional partial inductance models with respect to localization, we have shown that it is by far not as efficient as capacitance and application of pattern-matching techniques seem doubtful. Instead, we have described a windowing methodology based on field solutions within the localized regions, and how to combine the window solutions to produce a provably stable complete susceptance matrix. But since using susceptance directly in simulation also requires modification of simulation and circuit analysis tools, we have also described how to invert this complete susceptance matrix to and generate a sparse provably stable inductance matrix. Finally, we propose a modified magnetic vector potential definition that comes close to the accuracy of the double inverse results without requiring any matrix inversion.

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