Passive Interconnect Macromodeling Via Balanced Truncation of Linear Systems in Descriptor Form *

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ABSTRACT

In this paper, we present a novel passive model order reduction (MOR) method via projection-based truncated balanced realization method, *PriTBR*, for large RLC interconnect circuits. Different from existing passive truncated balanced realization (TBR) methods where numerically expensive Lur'e or algebraic Riccati (ARE's) equations are solved, the new method performs balanced truncation on linear system in descriptor form by solving generalized Lyapunov equations. Passivity preservation is achieved by congruence transformation instead of simple truncations. For the first time, passive model order reduction is achieved by combining Lyapunov equation based TBR method with congruence transformation. Compared with existing passive TBR, the new technique has the same accuracy and is numerically reliable, less expensive. In addition to passivity-preserving, it can be easily extended to preserve structure information inherent to RLC circuits, like block structure, reciprocity and sparsity. PriTBR can be applied as a second MOR stage combined with Krylov-subspace methods to generate a nearly optimal reduced model from a large scale interconnect circuit while passivity, structure, and reciprocity are preserved at the same time. Experimental results demonstrate the effectiveness of the proposed method and show PriTBR and its structure-preserving version, SP-PriTBR, are superior to existing passive TBR and Krylov-subspace based moment-matching methods.

Keywords

model order reduction, descriptor form, truncated balanced realization (TBR), passivity, structure

1. INTRODUCTION

Model order reduction (MOR) is an efficient technique to reduce the complexity of interconnect circuits while producing a good approximation of the input and output behavior.

Basically, there are two classes of MOR algorithms, namely, the Krylov-subspace based moment-matching algorithms [11, 2, 12, 7] and the recently promoted balanced truncation schemes [6, 8, 10, 14]. Starting with the modified nodal analysis (MNA) formulation of an interconnect circuit, Krylov-subspace based methods project the original system onto low rank subspace (Krylov-subspace) that captures most state activities at those frequencies of interest. Those approaches in general are scalable to reduce large VLSI interconnect circuits as projection matrices can be computed efficiently by solving linear equations. For RLC circuits, Krylov-subspace based methods like PRIMA [7] can also preserve the passivity of the original circuits and programs like SPRIM [3] can further preserve the structure information of circuit matrices like block structure, reciprocity, sparsity, etc, in addition to the passivity.

Krylov MOR techniques such as PRIMA, although is very suitable for analysis of large scale RLC circuits, do not necessarily generate models as compact as desired (that is, small in order for a given accuracy). There are no global error bounds between reduced model and original model. Therefore, another approach, truncated balanced realization (TBR), which has already well-developed in the control community [6], has been studied intensively recently.

In TBR method, two steps are involved in the reduction process: *balancing* step aligns the states such that states can be controlled and observed equally. The *truncating* step then throws away the weak states, which leads to a much smaller model. The major advantage of TBR methods is that TBR methods can give deterministic global bound for the approximate error and it can give nearly optimal models in terms of errors and model sizes [4].

Standard TBR algorithms, which solve Lyapunov equations (linear matrix equations) do not necessarily preserve passivity. To ensure the passivity of reduced model, positive-real TBR (PR-TBR) has to be carried out [9, 14] by solving more difficult Lur'e or Riccati equations, which can be computationally prohibitive as they are quadratic matrix equations.

Given a state-space model in *descriptor* form

$$\begin{aligned} E\frac{dx}{dt} &= Ax(t)+Bu(t) \\ y(t) &= Cx(t)+Du(t) \end{aligned} \qquad (1)$$

where $E, A \in R^{n\times n}, B \in R^{n\times p}, C \in R^{p\times n}, D \in R^{p\times p}, y(t), u(t) \in R^p$. When $E = I$, (1) is in standard state-space form. Note that the descriptor form is the natural form of the circuit MNA formulation for interconnect circuits, where E is the matrix of storage elements, A is the matrix of conductance and $B = C^T$ is the input and output position matrices and $D = 0$.

The existing passive TBR methods [9, 14] firstly convert original descriptor system into standard state-space system by mapping $E \rightarrow I$, $A \rightarrow E^{-1}A$, $B \rightarrow E^{-1}B$ and then solving two Lur'e or Riccati equations to guarantee the passivity of reduced model. However, there are several issues related to PR-TBR: First, it is not numerically reliable in the sense that given an ill-conditioned E, the mapping can generate too much numerical error and sometimes even the stability of the system can not be guaranteed in this process. Second, Lur'e and Riccati equations are quadratic matrix equations and thus more expensive than Lyapunov equations, which are linear matrix equations. Third, the structure property (block structure, reciprocity, sparsity) inherent to RLC circuits can not be preserved in the reduction process. Forth, in most cases, PR-TBR is not as accurate as standard TBR [9].

In this paper, we propose a novel passivity-preserving TBR method, named *PriTBR*, for interconnect modeling. Instead of working on standard state-space equations, PriTBR works on state-space equations in descriptor form directly by solving generalized Lyapunov equations. Due to the special matrix structure in descriptor form, congruence transformation can be applied to ensure the pas-

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sivity of reduced model. Compared with existing PR-TBR, which solves more difficult Lur'e or Riccati equations based on standard state-space model, the new method is numerical reliable, less expensive and more accurate. More important, it can be easily extended to preserve the structure information of RLC circuit matrices. Combined with structure-preserving Krylov-subspace based MOR methods, it can generate optimal structure-preserved reduced model from large-scale circuits.

This paper is organized as the following: Section 2 reviews the standard TBR methods and positive-real TBR methods. Section 3 presents our new passive TBR method. Section 4 gives a structure-preserving version of PriTBR. Experimental results are reported in Section 5 and Section 6 concludes the paper.

2. REVIEW OF TBR METHODS

In this section, we review standard balanced truncation methods [6] and positive real (passive) balanced truncation methods [9]. Both of them are working on a standard state space system ($E = I$ in (1)):

$$\begin{aligned} \frac{dx}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{2}$$

where $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{p\times n}$, $D \in R^{p\times p}$, $y(t)$, $u(t) \in R^p$.

2.1 Standard balanced truncation

Consider the system with A stable in standard state space form, the impulse response, $h(t) = Ce^{At}B, t \geq 0$, can be decomposed into an input-to-state map $x(t) = e^{At}B$, and a state-to-output map $\eta(t) = Ce^{At}$. Thus the input δ causes the state $x(t)$, while the initial condition $x(0)$ causes the output $y(t) = \eta(t)x(0)$. The grammians corresponding to x and η are

$$W_c = \int_0^\infty e^{At}BB^Te^{A^Tt}dt, \quad W_o = \int_0^\infty e^{A^Tt}C^TCe^{At}dt \tag{3}$$

which are the unique symmetric positive definite solutions to the Lyapunov equations.

$$AW_c + W_cA^T + BB^T = 0 \tag{4}$$

$$A^TW_o + W_oA + C^TC = 0 \tag{5}$$

The eigenvalues of the product W_cW_o are especially important because they contain information about the input-output behavior of the system: small eigenvalues correspond to internal sub-systems that have a weak effect on the input-output behavior of the system and are close to nonobservable or noncontrollable or both.

Since those eigenvalues are invariant under similarity transformation, we want to find a T to perform a similarity transformation to diagonalize the product W_cW_o

$$\tilde{W}_c\tilde{W}_o = T^{-1}W_cW_oT = diag(\sigma_1{}^2, \sigma_2{}^2, \ldots, \sigma_n{}^2) \tag{6}$$

Then the Hankel singular values of the system, σ, are the square roots of the eigenvalues of the product W_cW_o. After a transformation $x = T\tilde{x}$, the $\tilde{W}_c$ and $\tilde{W}_o$ are equal and diagonal and such a state space form is called *balanced*.

$$\tilde{W}_c = \tilde{W}_o = \Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_n) \tag{7}$$

The Hankel singular values characterize the 'importance' of state variables. States of the balanced system corresponding to the small Hankel singular values are difficult to reach and to observe at the same time. Such states are less involved in the energy transfer from inputs to outputs. Therefore, a general idea of balanced truncation is to transform system into a balanced form and to truncate the states that correspond to the small Hankel singular values. It turns out that every controllable and observable system can be transformed to a balanced form by means of a similarity transformation $x = T\tilde{x}$:

$$\tilde{A} = T^{-1}AT, \tilde{B} = T^{-1}B, \tilde{C} = CT, \tilde{D} = D, \tag{8}$$

We may partition Σ into

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{9}$$

Conformally partitioning the transformed matrices as

$$\tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \tilde{B} = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}, \tilde{C} = \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \end{bmatrix} \tag{10}$$

The reduced model of order q is obtained by simple truncation, i.e., by taking the $q\times q$, $q\times p$, $p\times q$ leading blocks of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, respectively; the system satisfies q-th order Lyapunov equations with diagonal solution Σ_1. This truncation leads to a balanced reduced-order system $(\tilde{A}_{11}, \tilde{B}_1, \tilde{C}_1, D)$.

Notice that the TBR method can also be viewed as a special projection method by projecting (2) onto the dominant eigenspace of the matrix W_cW_o corresponding to the largest eigenvalues, which motivates our projection-based TBR method.

2.2 Positive real balanced truncation

Standard TBR method does not ensure the passivity of the reduced models. To mitigate this problem, the following Lur'e equations are solved based on the positive real Lemma [1]: $H(s)$ is positive-real iff there exist matrices $X_c = X_c^T \geq 0$, J_c, K_c such that the Lur'e equations

$$\begin{aligned} AX_c + X_cA^T &= -K_cK_c^T \\ X_cC^T - B &= -K_cJ_c^T \\ J_cJ_c^T &= D + D^T \end{aligned} \tag{11}$$

are satisfied. And there exist matrices $X_o = X_o^T \geq 0$, J_o , K_o such that a dual set of Lur'e equations

$$\begin{aligned} A^TX_o + X_oA &= -K_o^TK_o \\ X_oB - C^T &= -K_o^TJ_o \\ J_o^TJ_o &= D + D^T \end{aligned} \tag{12}$$

are satisfied. X_c and X_o are analogous to the controllability grammian and observability grammian respectively. X_cX_o transforms under similarity just as W_cW_o so that their eigenvalues are invariant, and in fact in most respects they behave as the grammians W_c and W_o. We may find a coordinate system in which $\tilde{X}_c = \tilde{X}_o = \Sigma$, with Σ being again diagonal. In this coordinate system, the matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ may be partitioned and truncated, just as for the standard TBR procedure. The positive-real TBR (PR-TBR) was proposed to generate the passive models by solving the Lur'e equations [9].

The Lur'e equations can be transformed to Riccati equations, which are quadratic matrix equations and are more expensive to solve than Lyapunov equations, which are linear matrix equations.

3. NEW PASSIVE-PRESERVED BALANCED TRUNCATION

Our new approach is motivated by the recent mathematic work on balanced truncation for linear system in descriptor form [13] as shown in (1). Based on the MNA formulation of RLC circuits, we generalize this method so that both passivity and structure can be preserved in the reduction process.

In our PriTBR method, we directly work on systems in descriptor form, which is the natural form of RLC circuits in MNA formulation. Instead of obtaining a balanced form and truncating, we compute the basis, which spans the dominant subspace corresponding to the first q largest Hankel singular values and project the system onto the subspace so that the reduction process can be viewed as a congruence transformation. The difference between the new method and the standard TBR method is just like the difference between PRIMA [7] and Pade approximation via Lanczos (PVL) [2].

3.1 Projection based balanced truncation

Given a state-space model in descriptor form in (1) with the stable pencil $\lambda E - A$, which is usually the case in RLC circuit, we first assume E is non-singular. This restriction can be easily released with some additional steps [13]. If a system is in descriptor form, the controllable and observable grammians can be computed by solving generalized Lyapunov equations [13].

$$\begin{array}{l} EPA^T + APE^T + BB^T = 0 \\ E^T QA + A^T QE + C^T C = 0 \end{array} \quad (13)$$

The matrix $PE^T QE$ has nonnegative eigenvalues, and the square roots of these eigenvalues, $\sigma_j = \sqrt{\lambda_j(PE^T QE)}$, define the Hankel singular values of the system. Notice that we can also use PQ instead $PE^T QE$ as an alternative grammian product. We assume that the Hankel singular values are ordered decreasingly. System is called balanced if

$$P = Q = diag(\sigma_1, \sigma_2, \ldots, \sigma_n) \quad (14)$$

$PE^T QE$ here is similar to $W_c W_o$ in standard TBR method. After solving the generalized Lyapunov equations, one can compute the similarity transformation matrix T in (15) based on the square root method [5].

In this paper, instead of obtaining a balanced form and truncating, we perform reduction in a projection framework. As we know in standard TBR, the similarity transformation matrix T is the right eigenmatrix of $PE^T QE$

$$T^{-1} PE^T QET = diag({\sigma_1}^2, {\sigma_2}^2, \ldots, {\sigma_n}^2) \quad (15)$$

Therefore, the balancing and truncation can be viewed as a special projection of the system onto the dominant eigenspace of the matrix $PE^T QE$ corresponding to the q largest eigenvalues. After projection, the reduced model $(\tilde{E}, \tilde{A}, \tilde{B}, \tilde{C})$ is given by

$$\begin{array}{rcl} \tilde{E}\frac{dx}{dt} & = & \tilde{A}x(t) + \tilde{B}u(t) \\ y(t) & = & \tilde{C}x(t) + \tilde{D}u(t) \end{array} \quad (16)$$

where

$$\tilde{A} = W^T AV, \tilde{E} = W^T EV, \tilde{B} = W^T B, \tilde{C} = CV, \tilde{D} = D \quad (17)$$

$V_{n\times q}$ and $W_{n\times q}$ consist of the first q dominant eigenvectors of right eigenmatrix T and left eigenmatrix T^{-T}. Since two eigenmatrices cannot be the same $(V \neq W)$ for a general RLC circuit, the projection is an oblique projection and thus cannot guarantee the passivity of reduced model, which is just like Krylov-subspace based technique PVL [2].

3.2 Optimal subspace projection

In order to preserve the passivity of reduced system, an orthogonal projection has to be performed using one projection matrix like PRIMA [7]. In this paper, we use the right eigenmatrix T and partition T into

$$T = [\; T_1 \quad T_2 \;] \quad (18)$$

where $T_1 \in R^{n\times q}$, $T_2 \in R^{n\times(n-q)}$. Then the dominant subspace is spanned by basis T_1.

Although TBR-like methods are accurate globally, it cannot generate a reduced model with exact local behavior around DC as what is done in moment-matching based approaches, where an expansion point can be chosen right there. In this paper, we propose to add zero-order moment to the projection matrix. Since principle components under two optimal criteria are preserved at the same time, the reduced model has both global accuracy and exact low frequency behavior, which is much preferred. An orthonormalized composite projection matrix is given by

$$X = orth(M_0, T_1) \quad (19)$$

An orthogonal projection is performed to project the system onto the subspace spanned by X The reduced order system matrices are

$$\tilde{A} = X^T AX, \tilde{E} = X^T EX, \tilde{B} = X^T B, \tilde{C} = CX, \tilde{D} = D \quad (20)$$

These transformations are known as congruence transformations. In the following, we review congruence transformation and its passivity preserving property.

3.3 Passive reduction through congruence transformation

For RLC interconnect circuits, we can formulate the original circuit matrices with MNA formulation into a so-called passive form [7]

$$\begin{array}{rl} C\frac{dx}{dt} & = -Gx(t) + Bu(t) \\ y(t) & = B^T x(t) \end{array} \quad (21)$$

such that conductance matrix $G \geq 0$ and storage element matrix $C \geq 0$ are positive semi-definite and input and output position matrices are the same (notice that such passive form is also the descriptor form in (1)). It has been proved that the transfer function of system in such a passive form is positive real, meaning that the model is provably passive. The reduced matrices are

$$\tilde{C} = X^T CX, \; \tilde{G} = X^T GX, \; \tilde{B} = X^T B \quad (22)$$

Since congruence transformation preserves the definiteness of matrix, the reduced $\tilde{G}$, $\tilde{C}$ are still positive semi-definite. Then the transfer function of the reduced model will be positive real, and thus passive. Therefore, PriTBR can preserve the passivity for general RLC circuits.

For large systems, direct application of balanced truncation is computationally infeasible. Therefore, the methods are of more interest when combined with iterative Krylov-subspace procedures like PRIMA. Since an initial reduced model via PRIMA is also in this convenient passive form, PriTBR can always be used as the second stage of a composite model reduction procedure to generate a compact reduced model with provable passivity at a lower cost.

3.4 PriTBR reduction algorithm

A complete PriTBR reduction flow is given in Fig. 1. The basic algorithm is a generalization of the square root method used in standard TBR [5].

ALGORITHM 1: PROJECTION-BASED PASSIVE TBR (PRITBR)

1. Solve $EPA^T + APE^T + BB^T = 0$ for P
2. Solve $E^T QA + A^T QE + C^T C = 0$ for Q
3. Compute Cholesky factors $P = L_p L_p^T$, $Q = L_Q L_Q^T$
4. Compute SVD of $L_p^T E^T L_Q$:
   $$L_p^T E^T L_Q = [U_1, U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} [V_1, V_2]^T$$
5. Compute the dominant basis $T_1 = L_p U_1 \Sigma_1^{-1/2}$
6. Solve $AM_0 = B$ for M_0
7. Make a union of M_0 and T_1 and orthonormalize it $X = orth(M_0, T_1)$
8. Compute the reduced system with $\tilde{E} = X^T EX$; $\tilde{A} = X^T AX$; $\tilde{B} = X^T B$; $\tilde{C} = CX$

Figure 1: Projection-based passive TBR (PriTBR).

On top of PriTBR, we also propose the combined PriTBR and PRIMA flow to deal with large scale circuits, which is similar to the method in [4].

ALGORITHM 2: COMBINED PRIMA AND PRITBR MODEL ORDER REDUCTION

1. Perform PRIMA to get a small-size passivity-preserved initial reduced model from a large-size original model.
2. Perform PriTBR to get a optimal passivity-preserved final reduced model from the initial model.

Figure 2: Combined PRIMA and PriTBR methods.

3.5 Comparison with PR-TBR

Compared with passive PR-TBR method, this process has the following advantages:

First, we do not need to put a descriptor system into a standard form by mapping $E \to I$, $A \to E^{-1}A$, $B \to E^{-1}B$. After an initial projection (assuming Krylov-subspace method is performed as a first stage), E is usually nonsingular [9] but maybe ill-conditioned. As a result, the result by PR-TBR is no more accurate from numerical point of view. Even worse, sometimes, an unstable system can be generated after the mapping so that the Lur'e equations do not have positive semi-definite solutions.

Second, Lur'e equation is quadratic matrix equation, which is more expensive than linear matrix equation like Lyapunov equation. Generalized Lyapunov equation is still linear matrix equation and when E is nonsingular, it needs almost the same cost as Lyapunov equation [13].

Third, PriTBR has comparable accuracy as standard TBR while PR-TBR is usually not so accurate as standard TBR [9].

Fourth, as shown in the next section, in addition to passivity, PriTBR can be generalized to preserve block structure and reciprocity inherent to RLC circuits.

However, like PRIMA, it can not be used to systems outside the class of RLC circuits because both of them rely on congruence transformation to preserve passivity.

4. STRUCTURE-PRESERVED BALANCED TRUNCATION

While PR-TBR [9] generates provably passive reduced model, it does not preserve block structure or reciprocity, which are inherent to RLC circuits. However, the new approach, PriTBR, can be easily extended such that both passivity and structure can be preserved in a balanced truncation process, just like what is done in a moment-matching process SPRIM [3].

4.1 Structure preserved balanced truncation

Similar to SPRIM [3], we assume only current sources are applied and the transfer function is an impedance matrix function.

$$Z(s) = B^T(G+sC)^{-1}B \tag{23}$$

In MNA formulation of RLC circuits, G, C, and B have the block structure

$$G = \begin{bmatrix} G_1 & G_2^T \\ -G_2 & 0 \end{bmatrix}, C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \tag{24}$$

Let V be the union of matrices M_0 and T_1 used in PriTBR. Let

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{25}$$

be the partitioning of V corresponding to the block sizes of G and C. We then split the projection matrix V and orthonormalize each block respectively as

$$\tilde{V} = \begin{bmatrix} orth(V_1) & 0 \\ 0 & orth(V_2) \end{bmatrix} \tag{26}$$

Since

$$span(V) \subseteq span(\tilde{V}) \tag{27}$$

Therefore, we can project onto $\tilde{V}$, which spans subspace containing the subspace spanned by V. As shown in experiments, it has the good property of the standard TBR and matches the original model globally well.

At first glance, SP-PriTBR is not optimal in model size compared with PriTBR because given the same error bound, SP-PriTBR model would be twice as large as the corresponding PriTBR model. However, SP-PriTBR model can always be represented in second-order form. In this sense, the SP-PriTBR model (when written in second-order form) has the same dimension as PriTBR model given the same error bound.

$$\begin{gathered} \tilde{G_1} = V_1^T G_1 V_1, \tilde{G_2} = V_2^T G_2 V_1, \tilde{C_1} = V_1^T C_1 V_1 \\ \tilde{C_2} = V_2^T C_2 V_2, \tilde{B_1} = V_1^T B_1 \end{gathered} \tag{28}$$

and

$$\tilde{G} = \begin{bmatrix} \tilde{G_1} & \tilde{G_2}^T \\ -\tilde{G_2} & 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} \tilde{C_1} & 0 \\ 0 & \tilde{C_2} \end{bmatrix}, \tilde{B} = \begin{bmatrix} \tilde{B_1} \\ 0 \end{bmatrix} \tag{29}$$

So the block structure is preserved. The reduced model $\tilde{Z_n}$ in first-order form

$$\tilde{Z}(s) = \tilde{B}^T(\tilde{G}+s\tilde{C})^{-1}\tilde{B} \tag{30}$$

and in second-order form

$$\tilde{Z}(s) = \tilde{B}_1^T(s\tilde{C_1} + \tilde{G_1} + \frac{1}{s}\tilde{G}_2^T\tilde{C}_2^{-1}\tilde{G_2})^{-1}\tilde{B}_1 \tag{31}$$

and

$$\tilde{C_1} \succeq 0, \tilde{G_1} \succeq 0, \tilde{G}_2^T\tilde{C}_2^{-1}\tilde{G}_2 \succeq 0 \tag{32}$$

and thus $\tilde{Z}(s)$ is passive and also symmetric. This means that the reduced model preserves reciprocity and thus can be more easily synthesized as an actual circuit.

4.2 SP-PriTBR reduction algorithm

A complete SP-PriTBR reduction flow is given in Fig. 3. The SP-PriTBR can also work together with structure-preserving moment-matching based method like SPRIM to produce structure-preserved compact model from large scale circuits as shown in Fig. 4.

5. EXPERIMENTAL RESULTS

In this section, we show examples that illustrate the effectiveness of proposed PriTBR methods and compare it with existing relevant approaches. All the algorithms are implemented in Matlab 7.0

5.1 The accuracy of PriTBR

First, we demonstrate empirically that PriTBR has comparable accuracy as standard TBR. And compared with PRIMA, both of them are optimal in the sense that given the same reduced order, they are more accurate. Here, original model is a 515 order RC circuit stimulated by current sources. In Fig. 5, given the same reduced order 10, both PriTBR and standard TBR match equally well with the original curve and far better than PRIMA.

ALGORITHM 2: STRUCTURE-PRESERVING PRITBR ALGORITHM (SP-PRITBR)

1. Perform Algorithm2 (step 1- step 6) for $V = [M_0, T_1]$
2. Partition V corresponding to the block sizes of G, C
   $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
3. Set $\tilde{V} = \begin{bmatrix} orth(V_1) & 0 \\ 0 & orth(V_2) \end{bmatrix}$
4. Obtain the reduced model by projection:
   $\tilde{G} = \tilde{V}^T G\tilde{V}, \tilde{C} = \tilde{V}^T C\tilde{V}, \tilde{B} = \tilde{V}^T B$

Figure 3: Structure-preserving PriTBR algorithm

ALGORITHM 4: COMBINED SPRIM AND SP-PRITBR MODEL ORDER REDUCTION

1. Perform SPRIM to get a small-size structure-preserved initial reduced model from a large-size original model.
2. Perform SP-PriTBR to get an optimal structure-preserved final reduced model from the first step.

Figure 4: Combined SPRIM and SP-PriTBR methods.

5.2 The guaranteed passivity of PriTBR

The second example is a RLC transmission line (order 904) with voltage sources as input. The initial reduction is done by PRIMA and then followed by a TBR and PriTBR, respectively. The final reduced order is 12.

In Fig. 6, the Nyquist plots of the driving-point admittance is shown. This Nyquist plots contain both magnitude and phase information about the network admittance. It also provides a graphical test of port passivity. Indeed, it is well known that the Nyquist plots of positive real transfer functions lie entirely in the right half of the complex plane.

At first glance, it seems that reduced models are both passive. But when zooming in, we find that, for PriTBR, the entire Nyquist plot lies in the right half of the complex plane. However, for standard TBR, the Nyquist plot extends to the left half of the complex plane, which means that the passivity is not guaranteed in the reduced model.

5.3 The numerical reliability and accuracy of PriTBR

We use a RC circuit of 517 order with voltage sources as input to demonstrate the advantage of PriTBR. We do a initial reduction by PRIMA and get a reduced model of 75 order, which matches original curve very well.

First, if we want to use PR-TBR to do the second stage, we need to do the mapping $E \to I$, $A \to E^{-1}A$, $B \to E^{-1}B$ to put a descriptor system into a standard form. We find that after the mapping, the standard form is no more stable and thus no positive semi-definite solution is available for the following Lur'e equations. Fig. 7 show the pole zero maps of the system around origin before and after mapping. We can see after mapping a positive pole is generated.

Then, we employ PriTBR as the second reduction stage and get a final 15 order reduced model, which is still indistinguishable with the original curve as shown in Fig. 9. However, if we use PRIMA

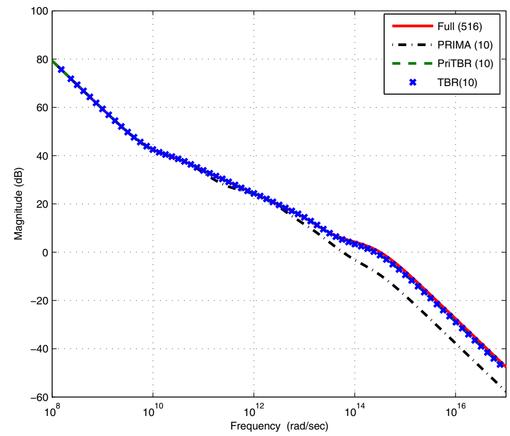

Figure 5: Frequency responses of TBR, PriTBR, PRIMA reduced models and original circuit.

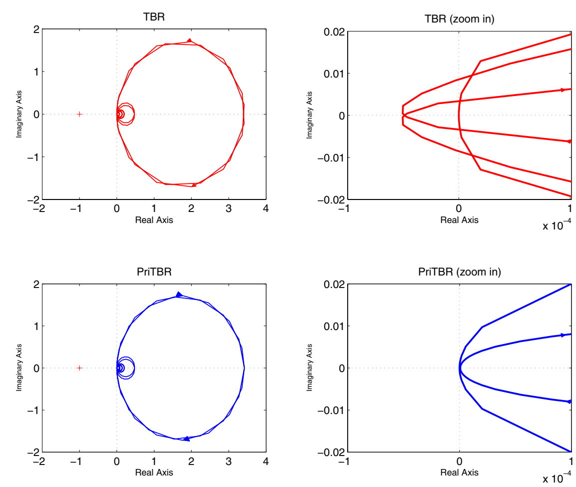

Figure 6: Nyquist plots of TBR reduced model and PriTBR reduced model.

to get a 15 order reduced model directly, we find the difference is so obvious.

5.4 The comparison of SPRIM and SP-PriTBR

We use a 302 order RLC circuit with current sources as input to compare SPRIM and SP-PriTBR with the same reduced order 10. The structure inherent to RLC circuit is preserved in both of them. We find SP-PriTBR inherits the optimal property of standard TBR and matches original curve better than SPRIM in a wide frequency band.

6. CONCLUSION

In this paper, we proposed a novel passive projection-based balanced truncation model reduction method, named *PriTBR*. The new method combines traditional TBR method with projection framework to produce passive models for the first time. It has both good error bounds from TBR method and passive reduction benefit from congruence transformation. Compared with existing passive TBR, the new technique is numerically reliable, more accurate and less expensive. In addition to passivity, PriTBR can be extended to

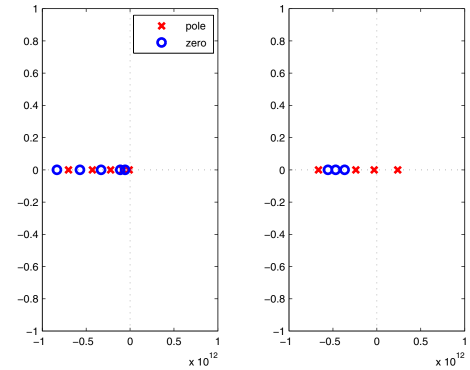

Figure 7: The pole-zero map of system before and after mapping.

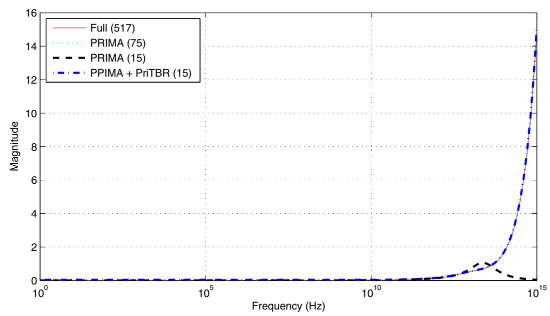

Figure 8: Frequency responses of PRIMA, composite (PRIMA and PriTBR) reduced models and original circuit.

preserve structure information like block structure and reciprocity. Combined with Krylov-subspace based approaches, it can be applied as a second stage of a composite MOR process to generate a nearly optimal reduced model for a large scale interconnect circuit while passivity, structure, and reciprocity can be preserved at the same time. Experimental results demonstrated the effectiveness of the proposed method and the advantage over existing passive TBR and Krylov-subspace based moment-matching methods.

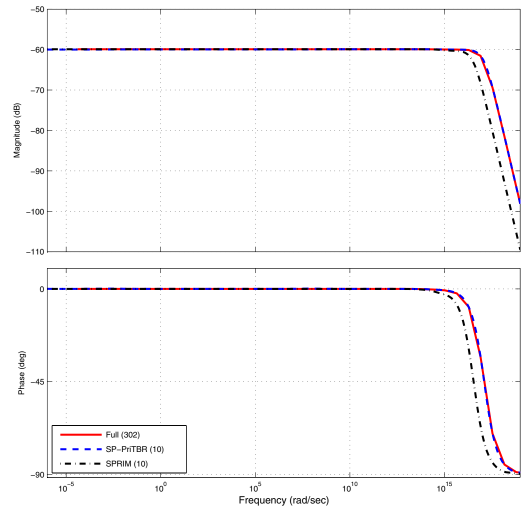

Figure 9: Frequency responses of SPRIM, SP-PriTBR reduced models and original circuit.

7. REFERENCES

[1] B. D. O. Anderson and S. Vongpanitlerd, *Network Analysis and Synthesis*. Englewood Cliffs, NJ: Prentice-Hall, 1973.

[2] P. Feldmann and R. W. Freund, "Efficient linear circuit analysis by pade approximation via the lanczos process," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 14, no. 5, pp. 639–649, May 1995.

[3] R. W. Freund, "SPRIM: structure-preserving reduced-order interconnect macromodeling," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2004, pp. 80–87.

[4] M. Kamon, F. Wang, and J. White, "Generating nearly optimially compact models from Krylov-subspace basded reduced-order models," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 47, no. 4, pp. 239–248, 2000.

[5] A. J. Laub, M. T. Heath, C. C. Paige, and R. C. Ward, "Computation of system balancing transformations and other applications of simultaneous diagonalization algorithms," *IEEE Trans. Automat. Contr.*, vol. AC-32, pp. 115–122, 1987.

[6] B. Moore, "Principle component analysis in linear systems: Controllability, and observability, and model reduction," *IEEE Trans. Automat. Contr.*, vol. AC-26, no. 1, pp. 17–32, 1981.

[7] A. Odabasioglu, M. Celik, and L. Pileggi, "PRIMA: Passive reduced-order interconnect macromodeling algorithm," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, pp. 645–654, 1998.

[8] J. R. Phillips, L. Daniel, and L. M. Silveira, "Guaranteed passive balancing transformations for model order reduction," in *Proc. Design Automation Conf. (DAC)*, 2002, pp. 52–57.

[9] ——, "Guaranteed passive balanced transformation for model order reduction," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 22, no. 8, pp. 1027–1041, 2003.

[10] J. R. Phillips and L. M. Silveira, "Poor man's TBR: a simple model reduction scheme," in *Proc. European Design and Test Conf. (DATE)*, 2004, pp. 938–943.

[11] L. T. Pillage and R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, pp. 352–366, April 1990.

[12] M. Silveira, M. Kamon, I. Elfadel, and J. White, "A coordinate-transformed Arnoldi algorithm for generating guaranteed stable reduced-order models of RLC circuits," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 1996, pp. 288–294.

[13] T. Stykel, "Gramian-based model model reduction for descriptor systems," *Math. Control Signals Systems*, vol. 16, pp. 297–319, 2004.

[14] N. Wang and V. Balakrishnan, "Fast balanced stochastic trucation via a duadratic extension of the alternating direction implicit iteration," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2005, pp. 801–805.